



# Low-Frequency Rotational Seismology: Observations with Ring Laser Technology

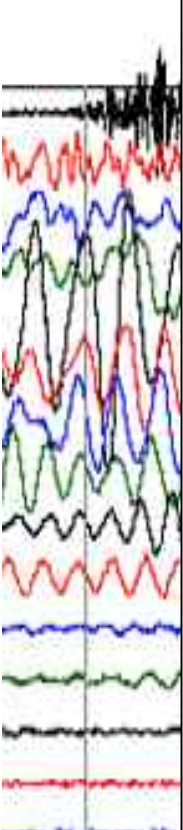
H. Igel<sup>1</sup>, A. Cochard<sup>1</sup>, A. Fichtner<sup>1</sup>, A. Flaws<sup>1,2</sup>, U. Schreiber<sup>3</sup>, B. Schuberth<sup>1</sup>, W. Suryanto<sup>1</sup>, D.N. Pham<sup>1</sup>, A. Velikoseltsev<sup>3</sup>, F. Vernon<sup>4</sup>, J. Wassermann<sup>1</sup>, M. Bernauer<sup>1</sup>, F. Bernauer<sup>1</sup>, D. Kurrle<sup>1</sup>

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<sup>3</sup>Fundamentalstation Wettzell, Kötzing

<sup>4</sup>IGPP, La Jolla, USA

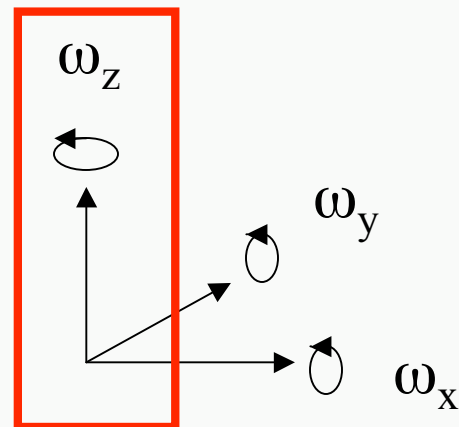


- What is *rotation* in seismology? (Why bother?)
- The ring laser instrument
- Observations: What to do with rotations?
  - Waveform comparison with translations
  - Tomography without travel times
  - **Free oscillations of the Earth!!!**
- Array-derived vs. directly measured rotations
- Conclusions and Outlook

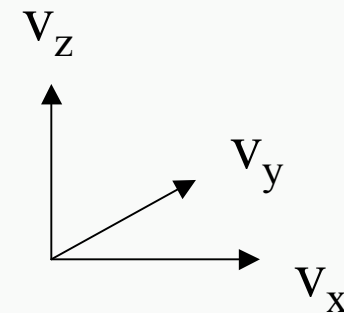
# Rotation is the **curl** of the wavefield

... it separates P- and S-waves in isotropic media

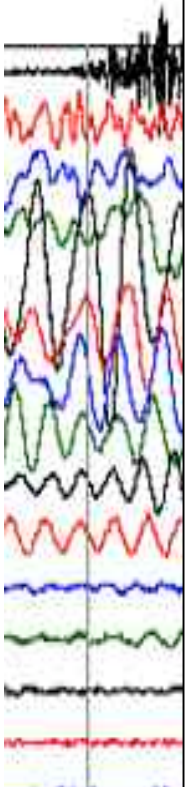
$$\begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \frac{1}{2} \nabla \times \underline{\mathbf{v}} = \frac{1}{2} \begin{pmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \partial_x v_y - \partial_y v_x \end{pmatrix}$$



Rotation rate  
***Rotation sensor***

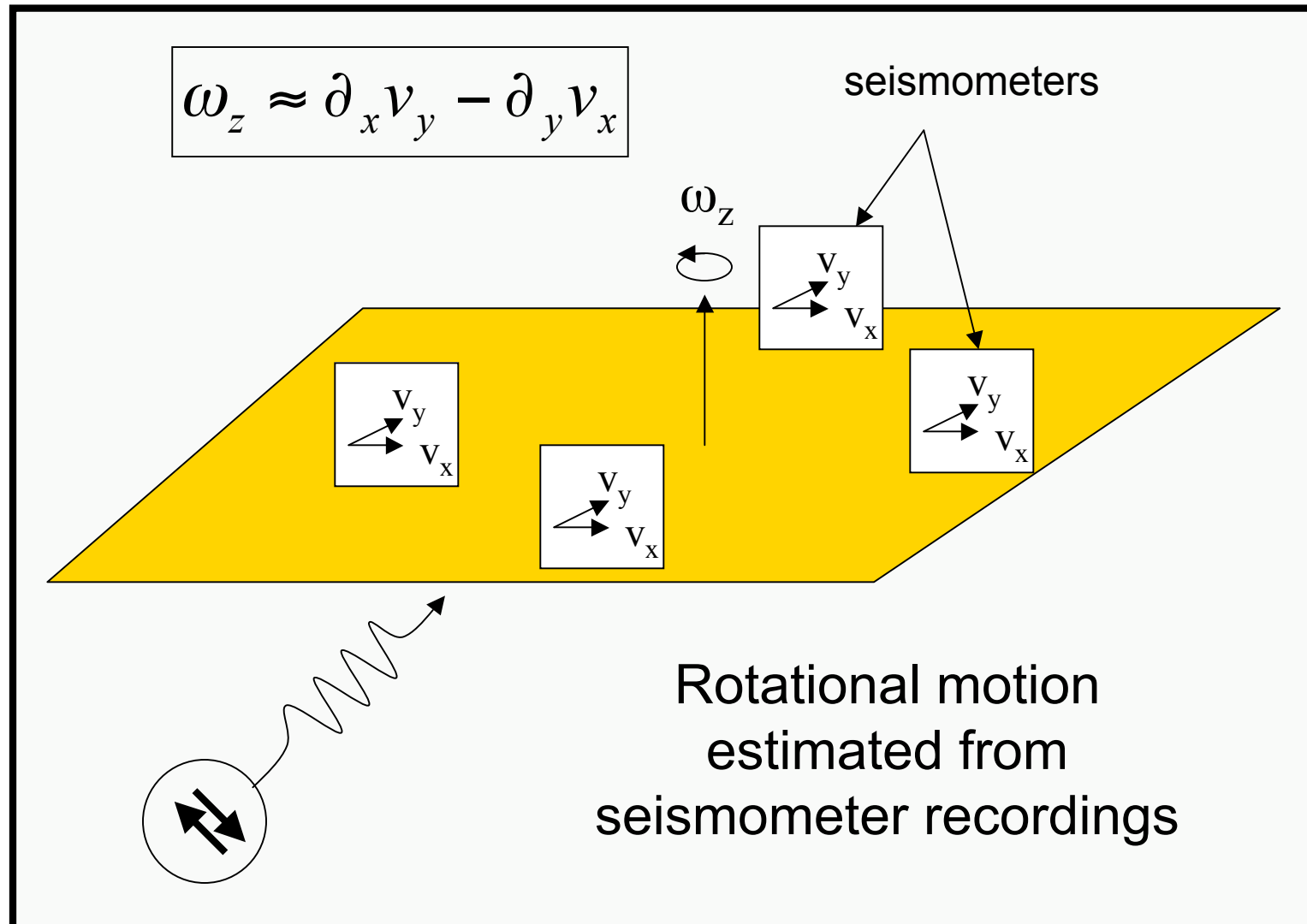
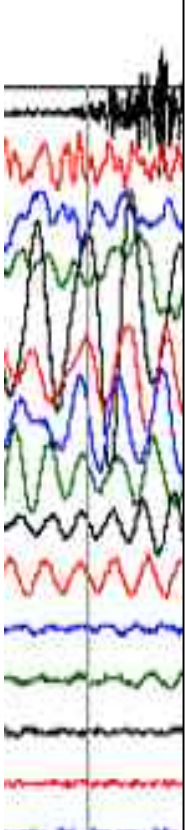


Ground velocity  
***Seismometer***

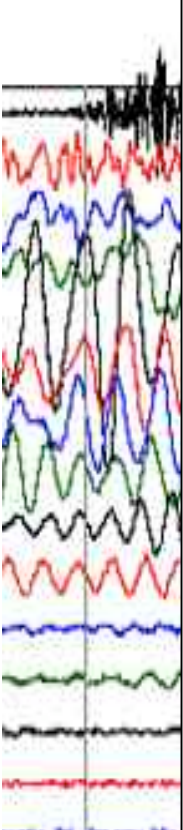


# Rotation from seismic arrays?

... by finite differencing ...



# Rotations - why bother?

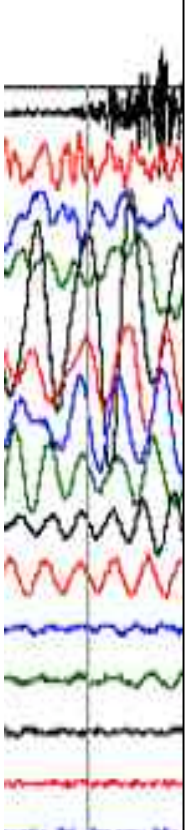


- Standard seismological observations are **contaminated** by rotations (e.g. OBS, strong motions, free oscillations)
- Tiltmeters (rotation around horizontal axes) are **contaminated** by translations
- 6C sensor may allow **integration to displacements**
- Rotations may contribute to **co-seismic structural damage**
- Rotational measurements provides additional wavefield information (**phase velocities, structure, propagation direction, anisotropy**, etc)
- ... and may allow putting further constraints on **rupture processes** ...

Instruments

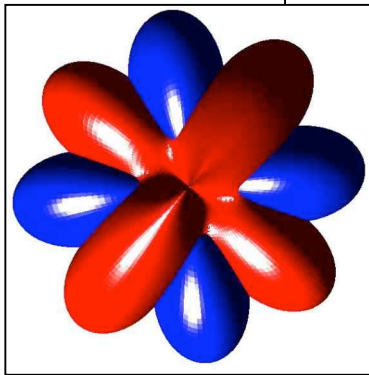
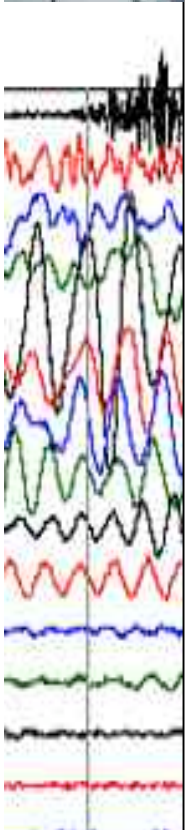
Earthquake engineering

Tomography and sources



THEORY

# Radiation from a point source



Ground displacement

$$\begin{aligned}
 u(x, t) = & \frac{1}{4\pi\rho} A^N \frac{1}{r^4} \int_{r/v_P}^{r/v_S} \tau M_0(t - \tau) d\tau \\
 & + \frac{1}{4\pi\rho v_P^2} A^{IP} \frac{1}{r^2} M_0(t - r/v_P) \\
 & + \frac{1}{4\pi\rho v_S^2} A^{IS} \frac{1}{r^2} M_0(t - r/v_S) \\
 & + \frac{1}{4\pi\rho v_P^3} A^{FP} \frac{1}{r} \dot{M}_0(t - r/v_P) \\
 & + \frac{1}{4\pi\rho v_S^3} A^{FS} \frac{1}{r} \dot{M}_0(t - r/v_S).
 \end{aligned}$$

Near field term contains the static displacement

Intermediate terms

Far field terms: the main ingredient for source inversion, ray theory, etc.

Aki and Richards (2002)



# The rotational part

$$\begin{aligned}\omega(\mathbf{x}, t) &= \frac{1}{2} \nabla \times \mathbf{u}(\mathbf{x}, t) \\ &= \frac{-\mathbf{A}^R}{8\pi\rho} \left[ \frac{3}{\beta^2 r^3} M_0 \left( t - \frac{r}{\beta} \right) + \frac{3}{\beta^3 r^2} \dot{M}_0 \left( t - \frac{r}{\beta} \right) + \frac{1}{\beta^4 r} \ddot{M}_0 \left( t - \frac{r}{\beta} \right) \right]\end{aligned}$$

$$\mathbf{A}^R = \cos \theta \sin \phi \hat{\boldsymbol{\theta}} + \cos \phi \cos 2\theta \hat{\boldsymbol{\phi}}$$

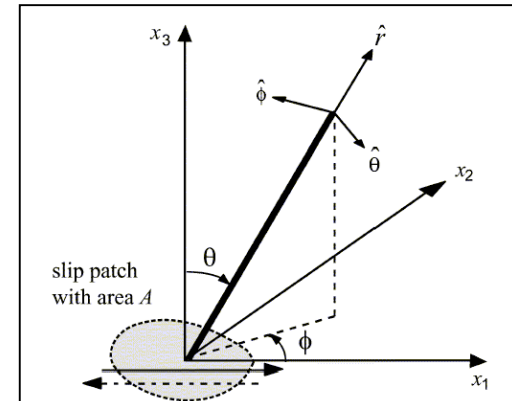
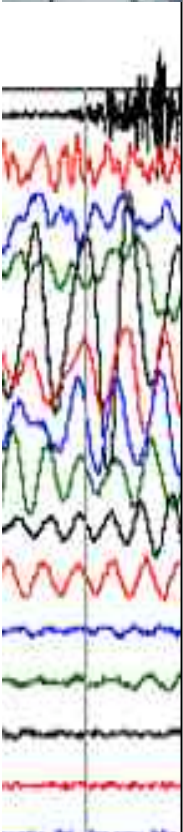


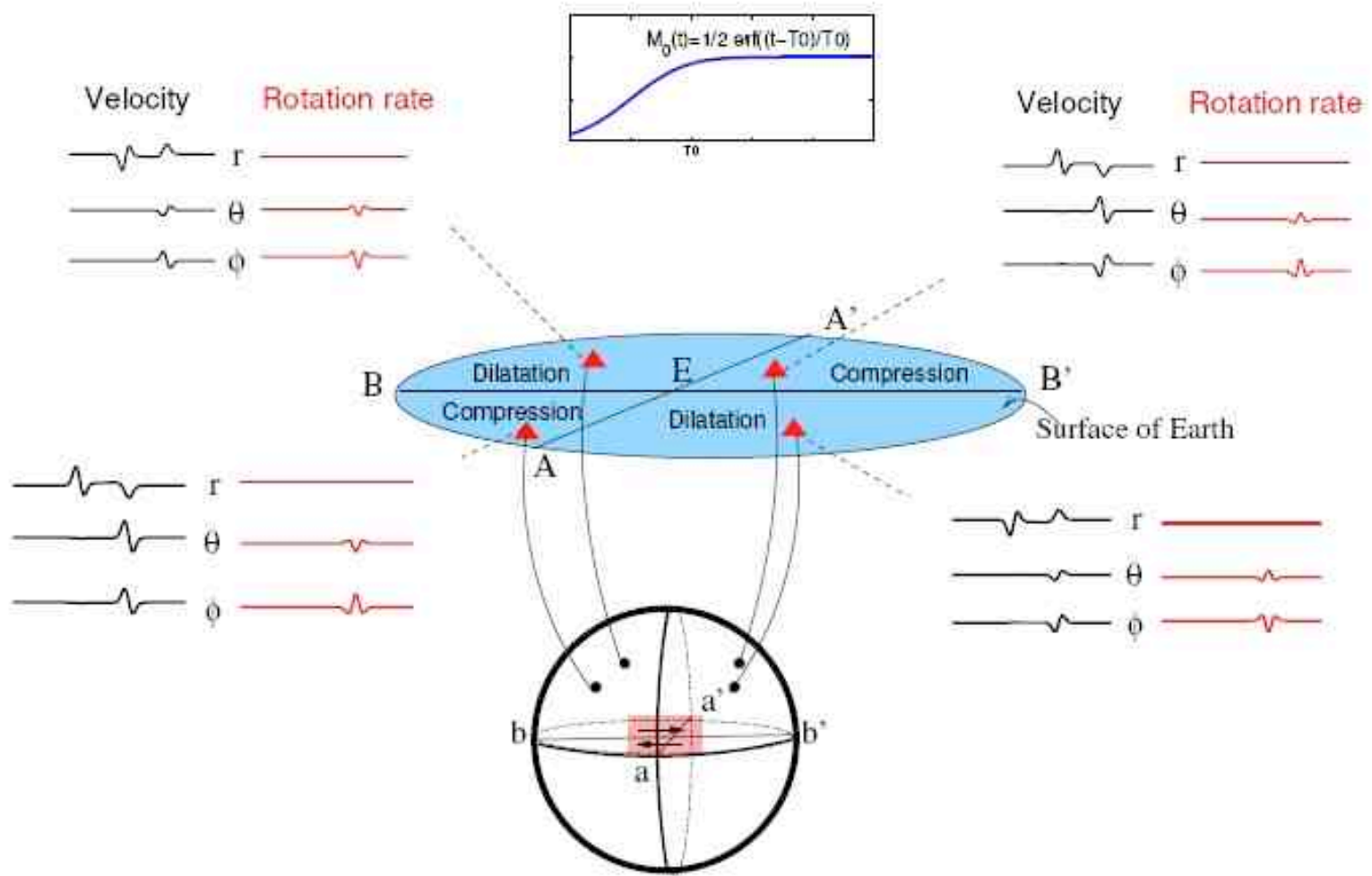
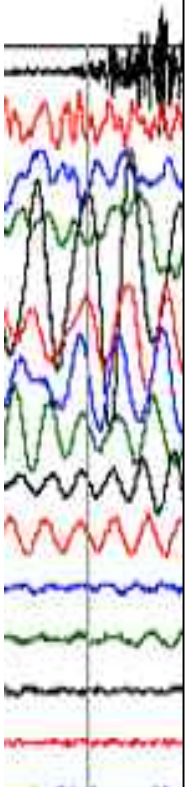
FIGURE 5 Cartesian and polar coordinate systems for analysis of radiation by a slip patch with area  $A$  and average slip  $\langle \Delta u(t) \rangle$ .

- Rotations are zero before  $S$  arrival
- Far-field P-rotation is not zero! Only the sum of all contributions cancel!

Cochard et al. (2006)



# Basic seismograms, full space







# Rotation rate and transverse acceleration plane-wave propagation

Plane transversely polarized wave propagating in x-direction with phase velocity  $c$

$$u_y(x, t) = f(kx - \omega t) \quad c = \omega / k$$

Acceleration

$$a_y(x, t) = \ddot{u}_y(x, t) = \omega^2 f''(kx - \omega t)$$

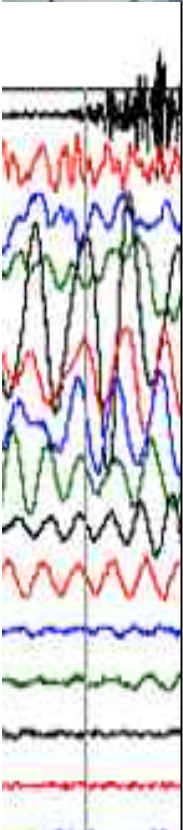
Rotation rate

$$\dot{\Omega}(x, t) = \frac{1}{2} \nabla \times [0, \dot{u}_y, 0] = \left[ 0, 0, -\frac{1}{2} k \omega f''(kx - \omega t) \right]$$



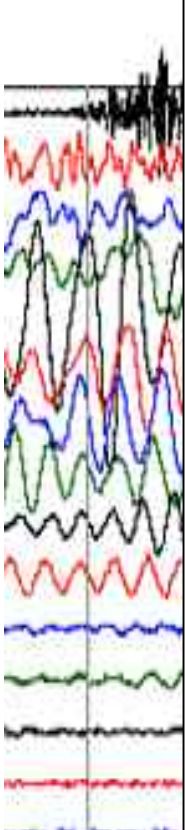
$$a(x, t) / \dot{\Omega}(x, t) = -2c$$

Rotation rate and acceleration should be **in phase** and the **amplitudes scaled by two times the horizontal phase velocity**

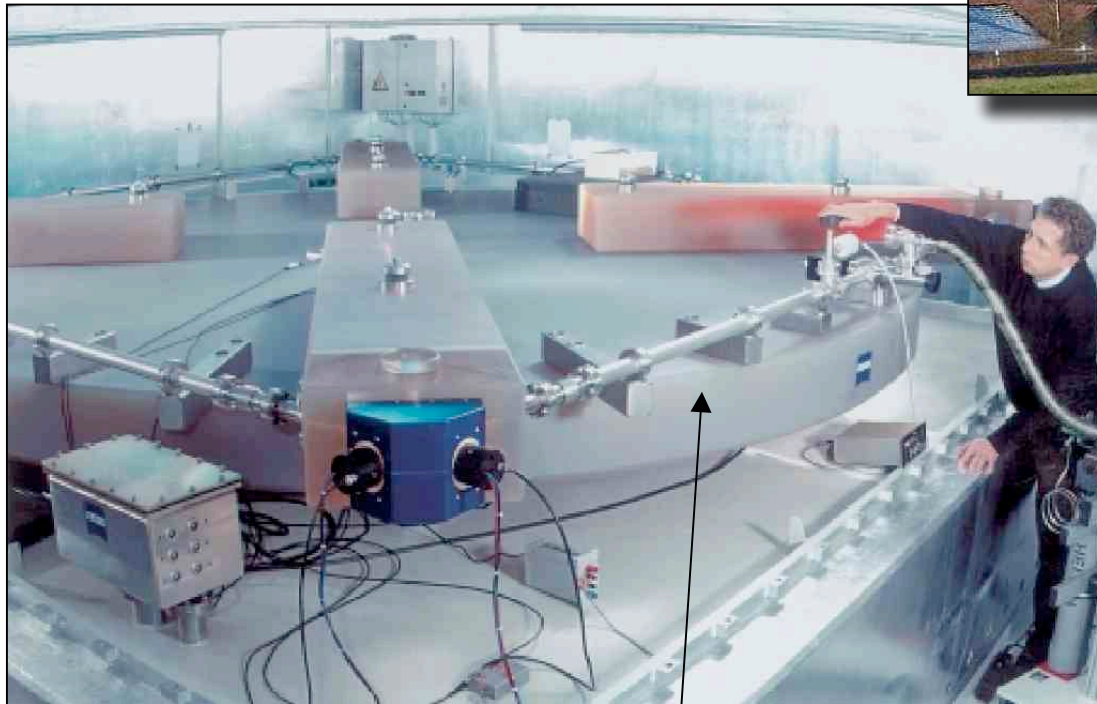
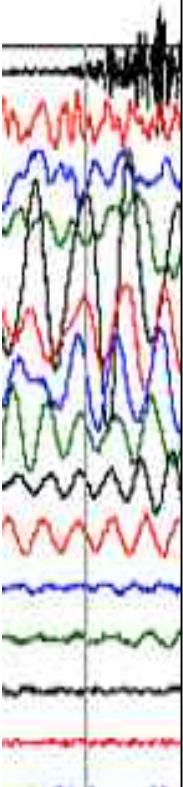




# Instrumentation

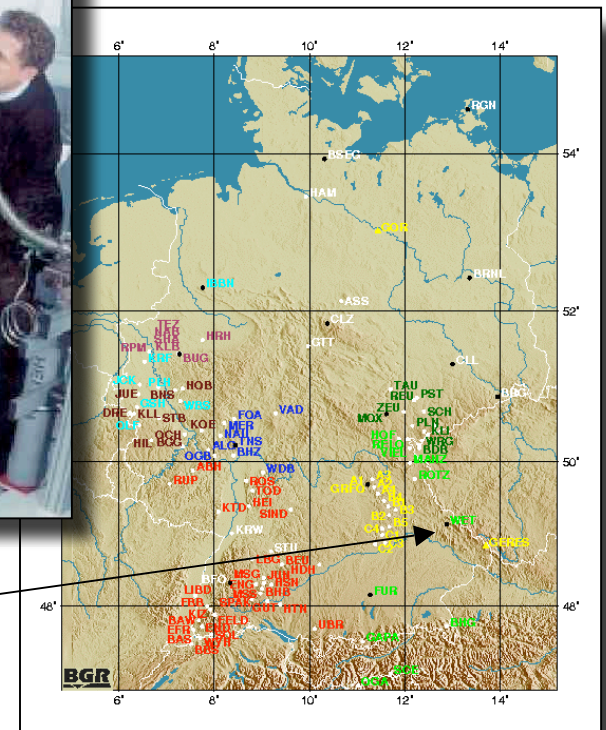


# The ring laser at Wettzell



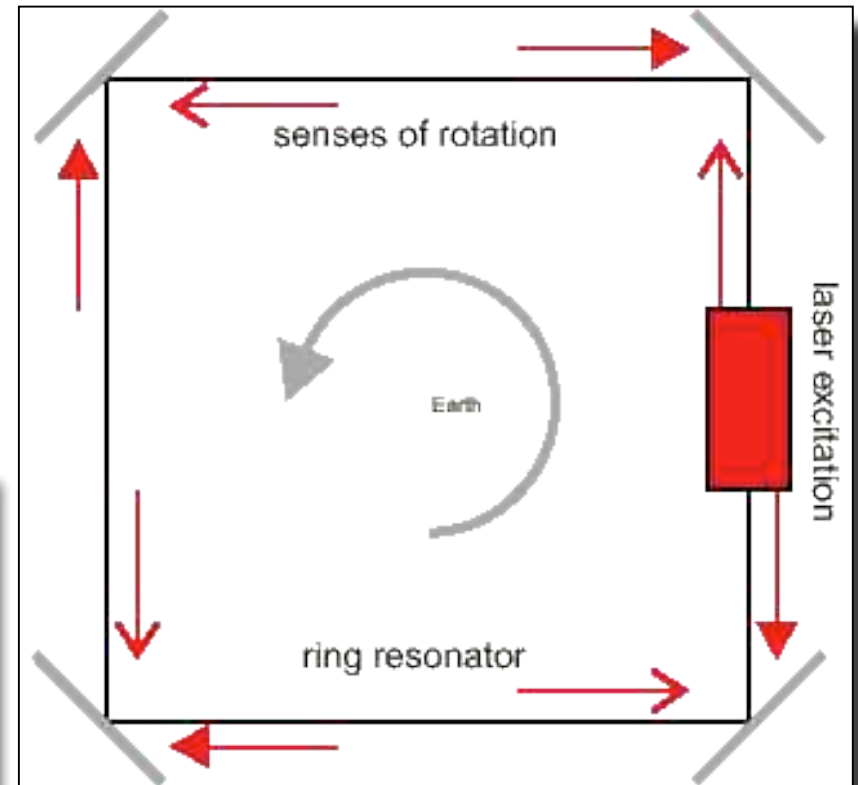
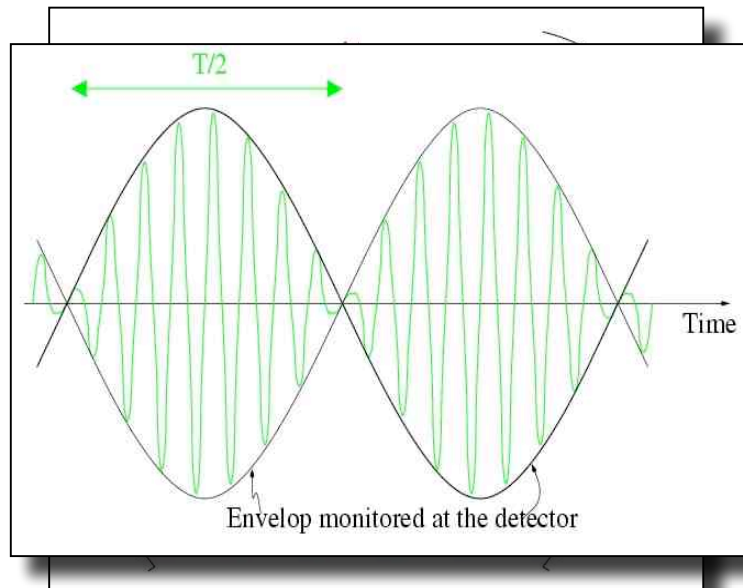
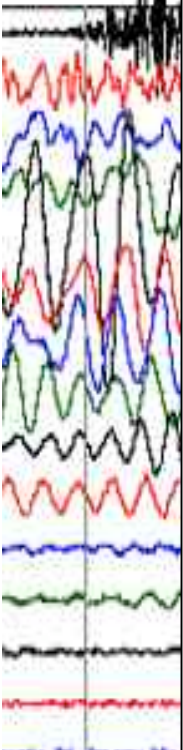
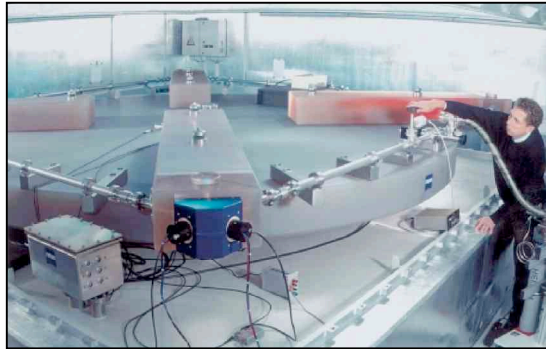
ring laser

Data accessible at [www.rotational-seismology.org](http://www.rotational-seismology.org)



# How can we observe rotations?

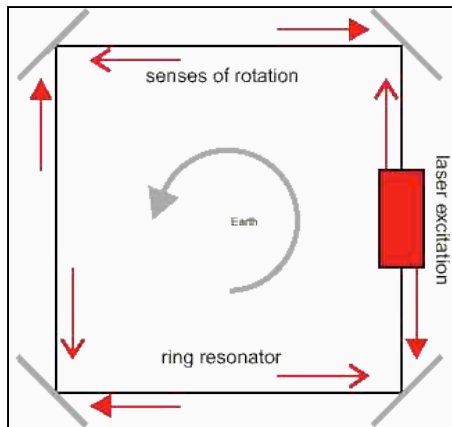
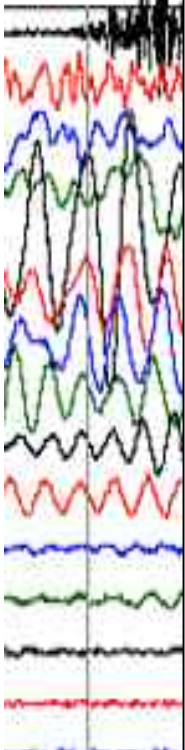
-> ring laser



Ring laser technology developed by the groups at the Technical University Munich and the University of Christchurch, NZ



# Ring laser - the principle

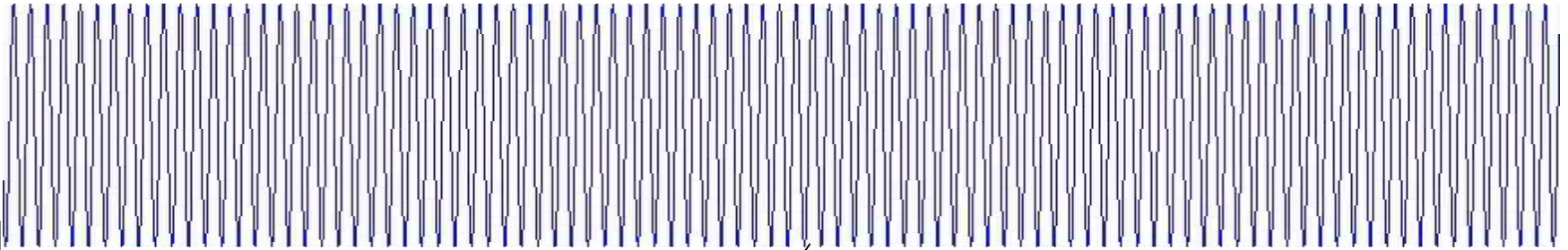
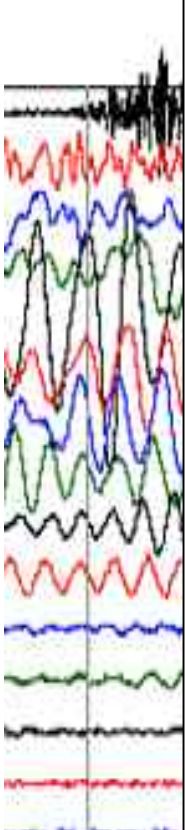


$$\Delta f_{Sagnac} = \frac{4\dot{\mathbf{U}} \cdot \mathbf{A}}{\lambda P}$$

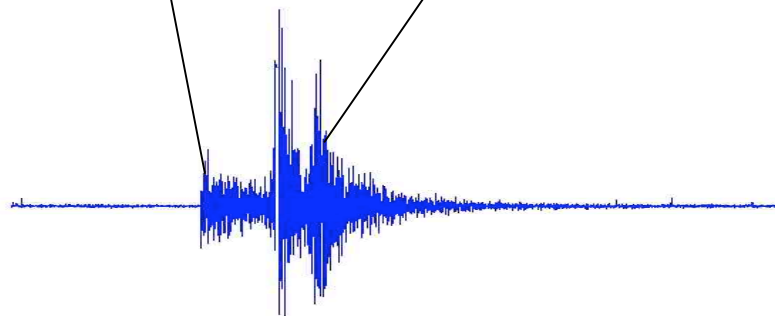
- $\mathbf{A}$  surface of the ring laser (vector)
- $\Omega$  imposed rotation rate (Earth's rotation + earthquake +...)
- $\lambda$  laser wavelength (e.g. 633 nm)
- $P$  perimeter (e.g. 4-16m)
- $\Delta f$  Sagnac frequency (e.g. 348,6 Hz sampled at 1000Hz)

Resolution down to  $O(10^{-11})$  rad/s

# The Sagnac Frequency (schematically)



Sagnac frequency sampled with 1000Hz (Reftek)  
→ instantaneous freq. using Hilbert transform



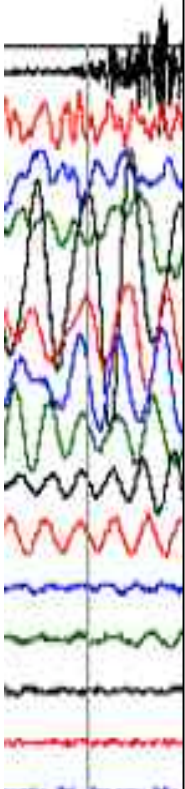
Tiny changes in the Sagnac frequencies are extracted to obtain the time series with rotation rate  
 $\Delta f \rightarrow \Theta$

Rotation rate sampled with 20Hz

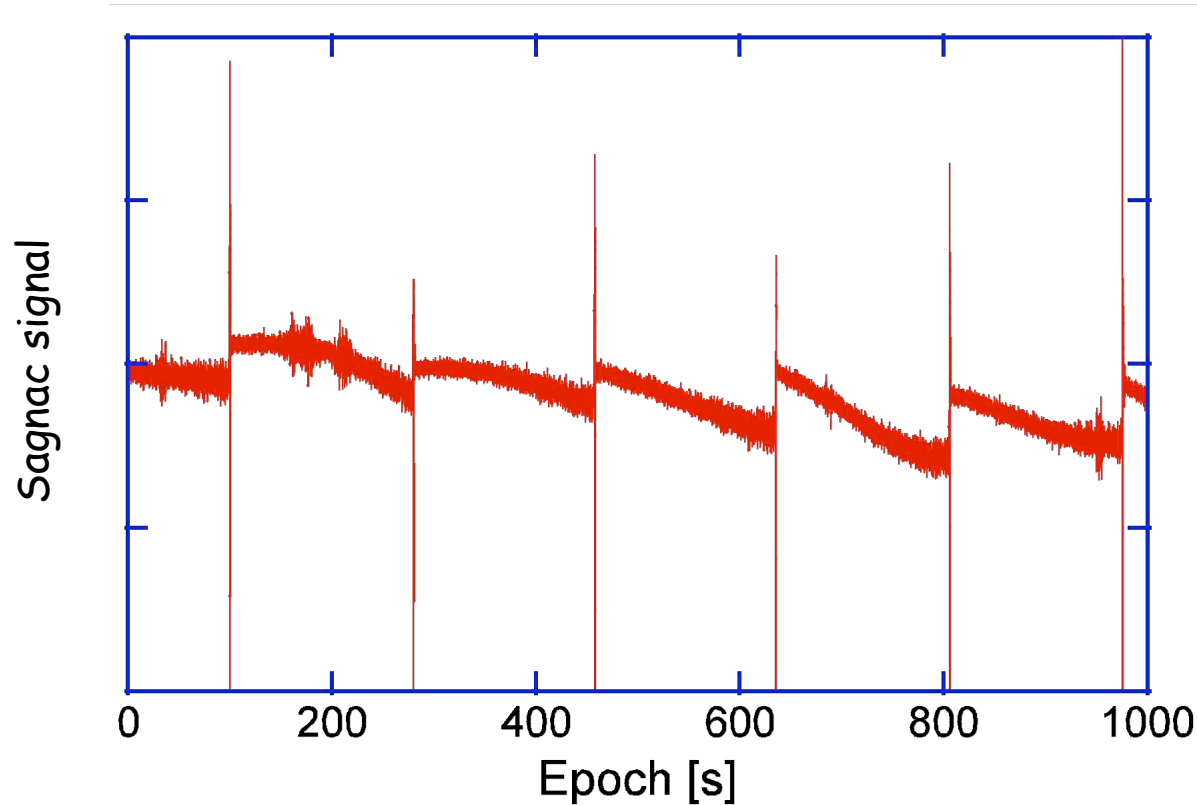


# The PFO sensor

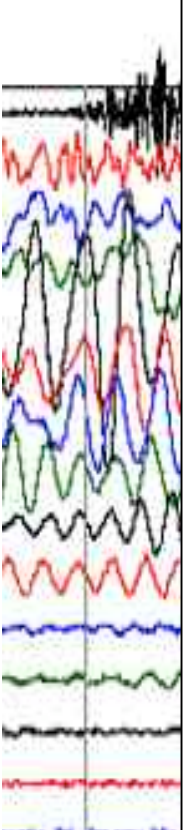
... built for seismology



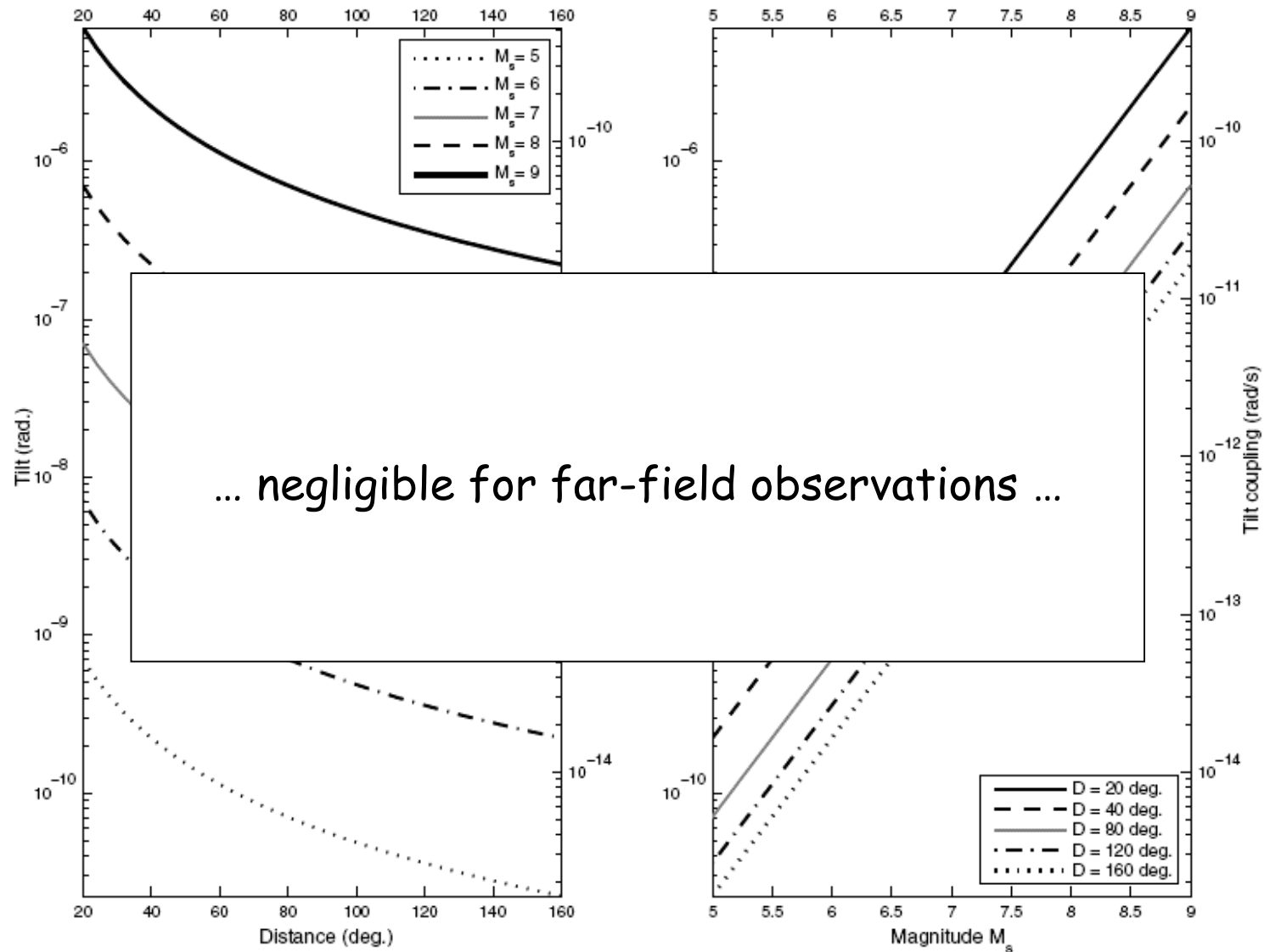
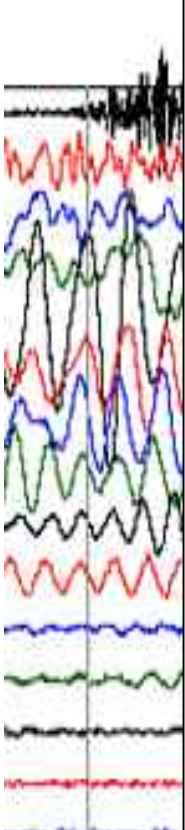
# The PFO sensor mode hopping



... mode hops can be avoided by cleaning through *getter function* (reducing outgassing)



# Cross-axis sensitivity: tilt-Ringlaser coupling



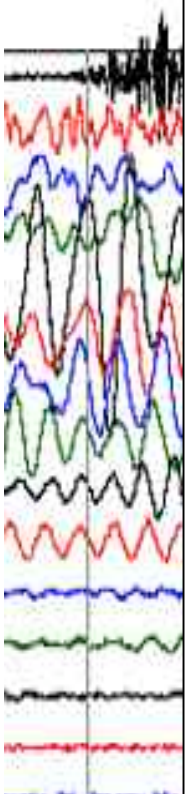


# Horizontal axes?

- First application in VIRGO project (gravitational wave detection, Pisa, Italy)
- 3C planned for PFO
- Options to get around „lock-in“ problem:
  - 45 degree arrangement, projections
  - Running RL with „dithering“ out of lock-in

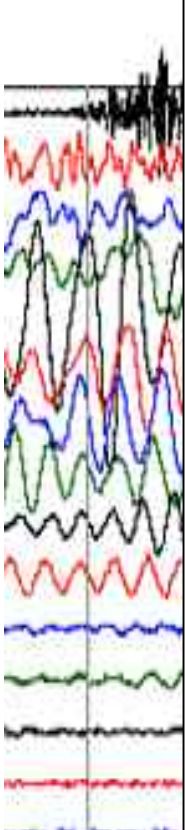
RL1

RL2



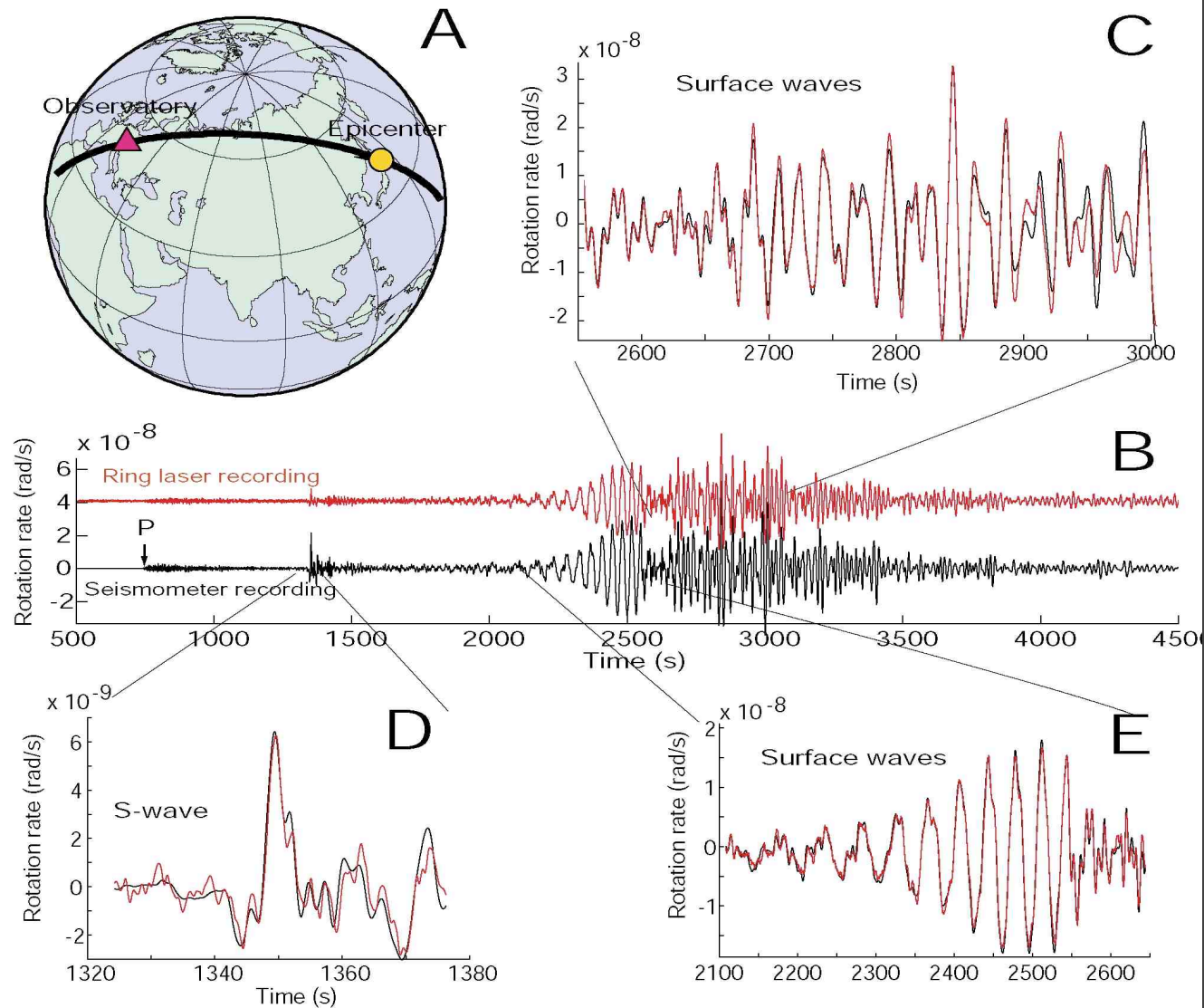
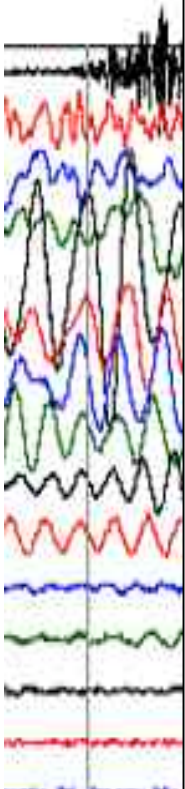


Data, Data, Data



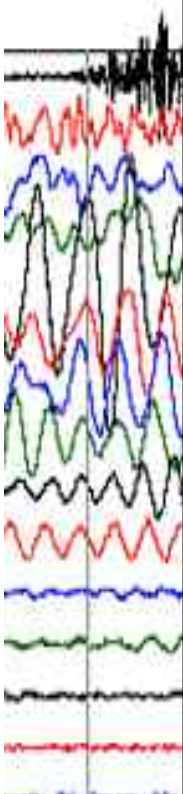
# Mw = 8.3 Tokachi-oki 25.09.2003

## transverse acceleration - **rotation rate**



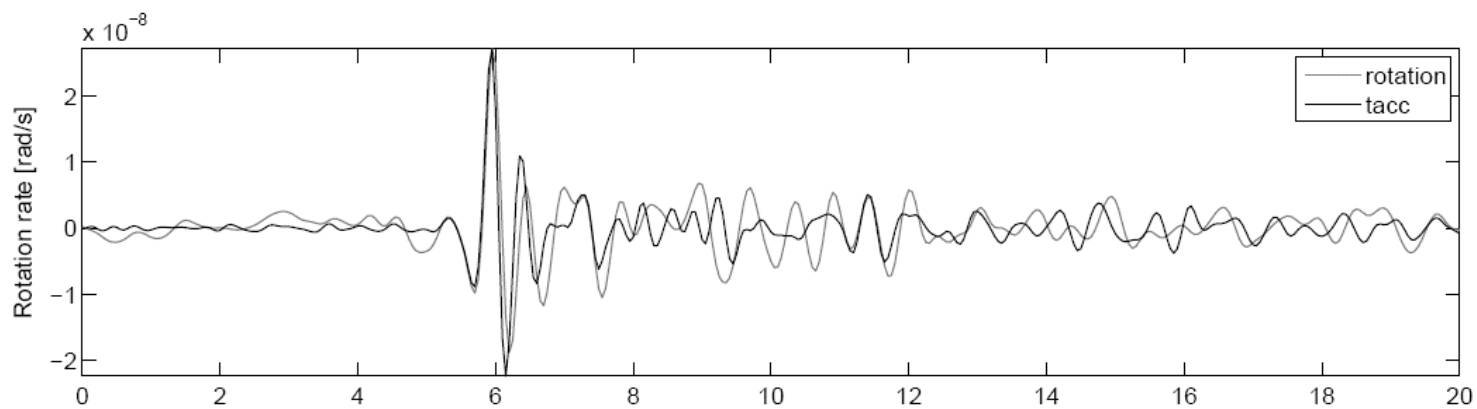
From Igel et al., GRL, 2005



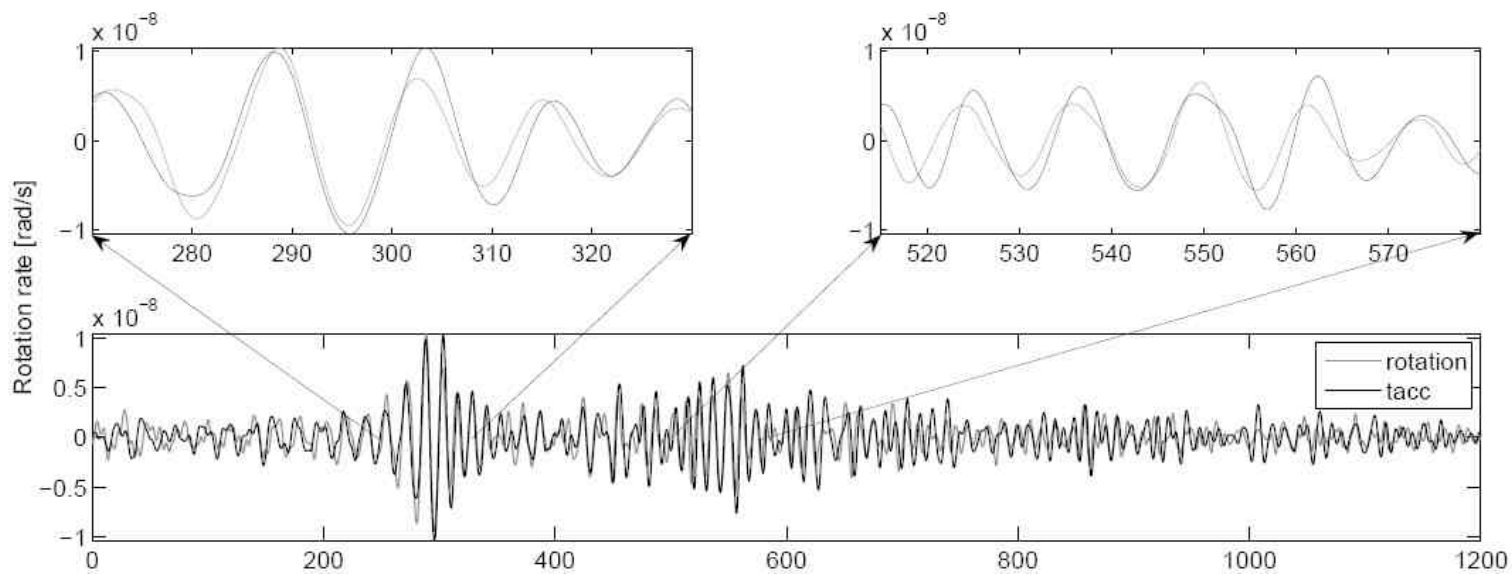


# PFO Observations

California M3.9



Kamtchatka M7.6

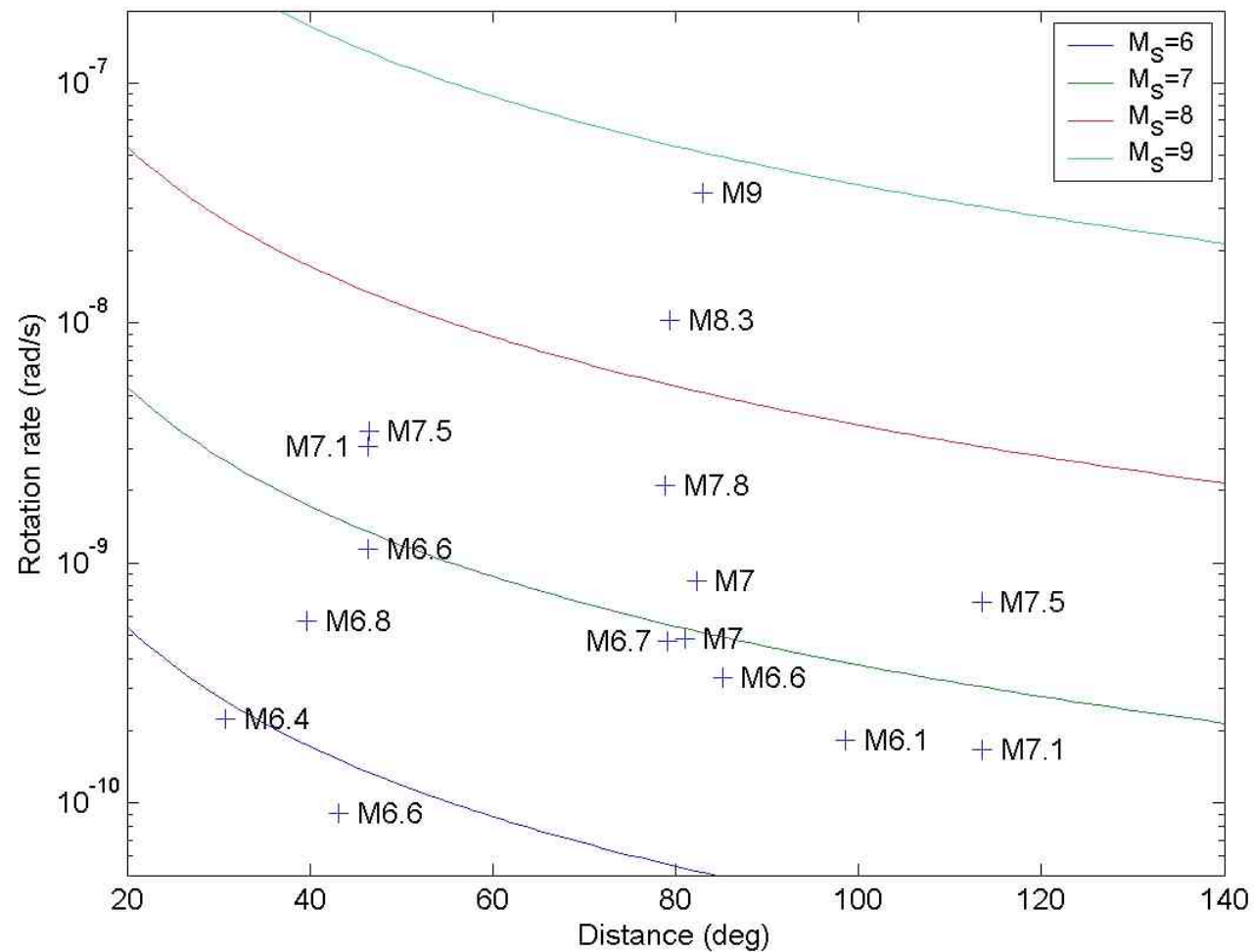
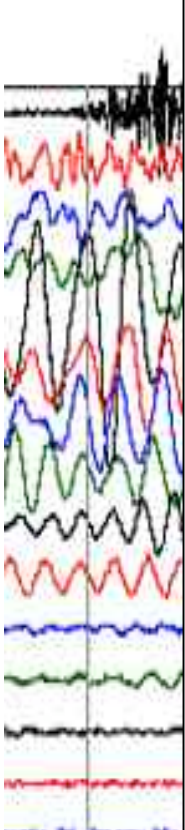




Compatibility with  $M_S$   
(surface wave magnitude)  
 $T=30s$ ,  $c=4300m/s$

$$M_S = \log_{10} \frac{A}{T} + 1.66 \log_{10} D + 3.3$$

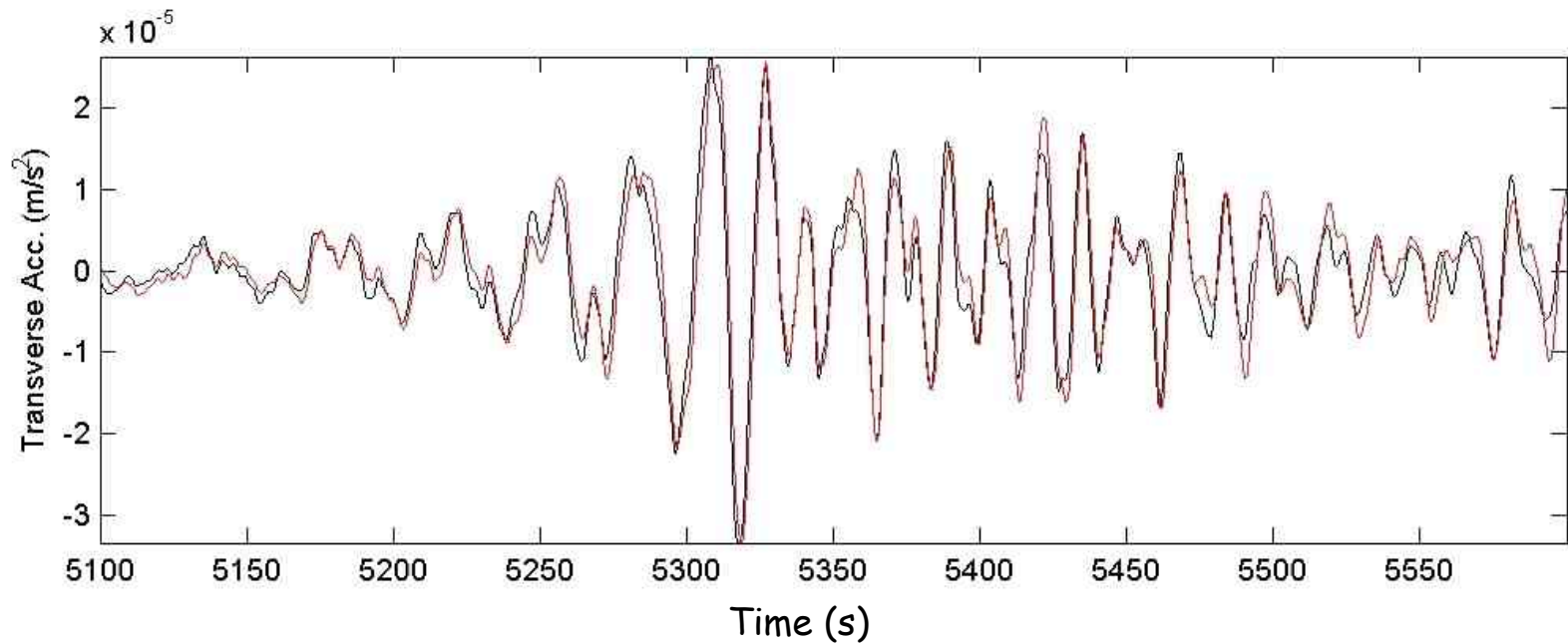
$$\Omega_z = 2 \frac{\pi^2}{cT^2} A(M_S, D) = 2 \frac{\pi^2}{cT} 10^{M_S - 1.66 \log_{10} D - 9.3}$$



# Instrument correction!

## transverse acceleration - **rotation rate**

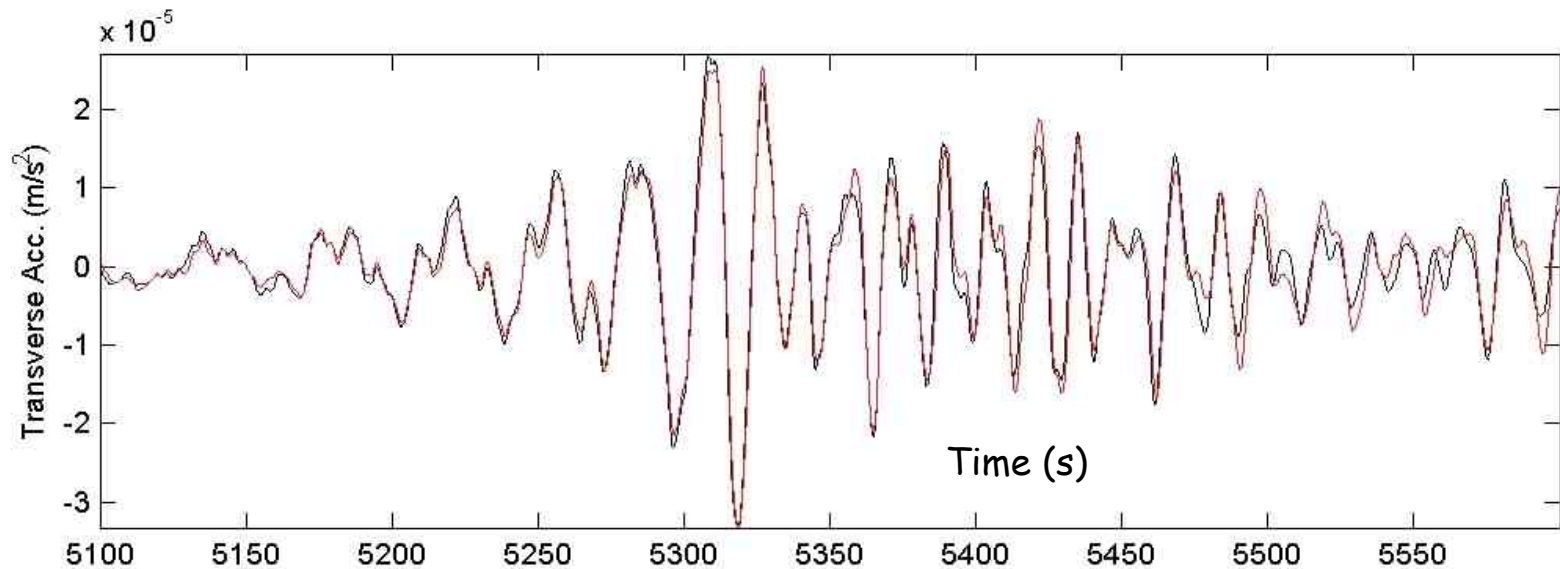
Before correction



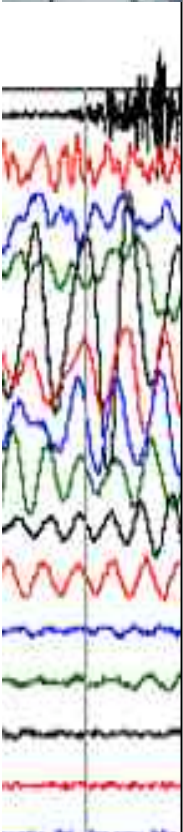
# Instrument correction!

## transverse acceleration - rotation rate

After correction

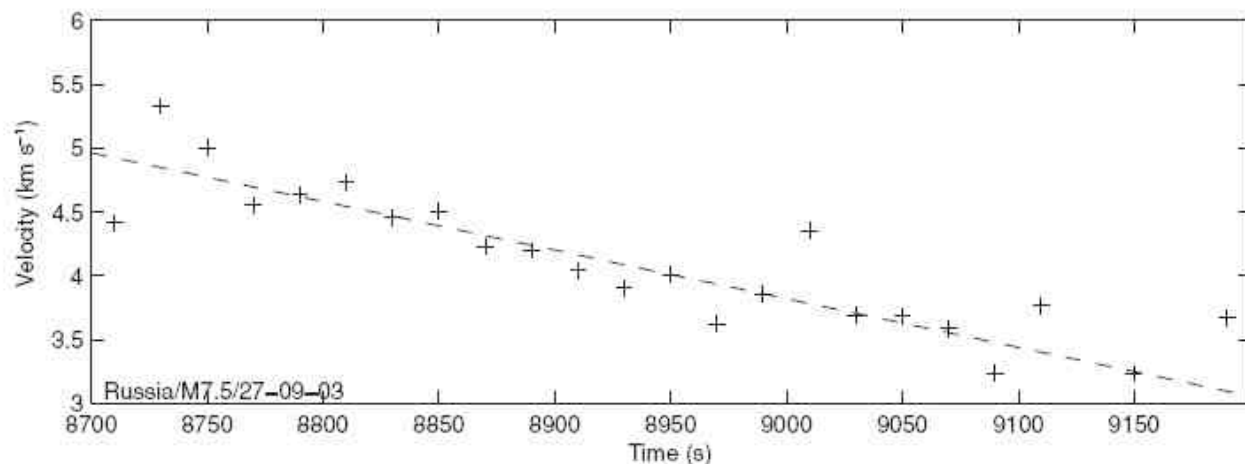
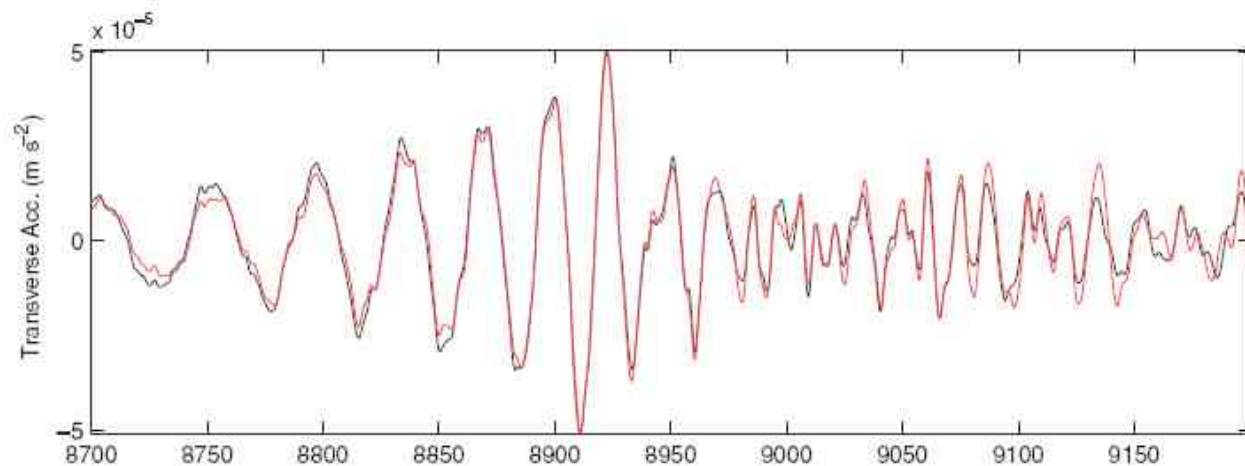
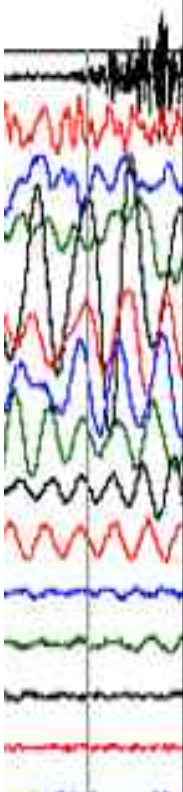


... an independent confirmation of the quality of  
the instrument correction ...



# Love wave dispersion

transverse acceleration - **rotation rate**

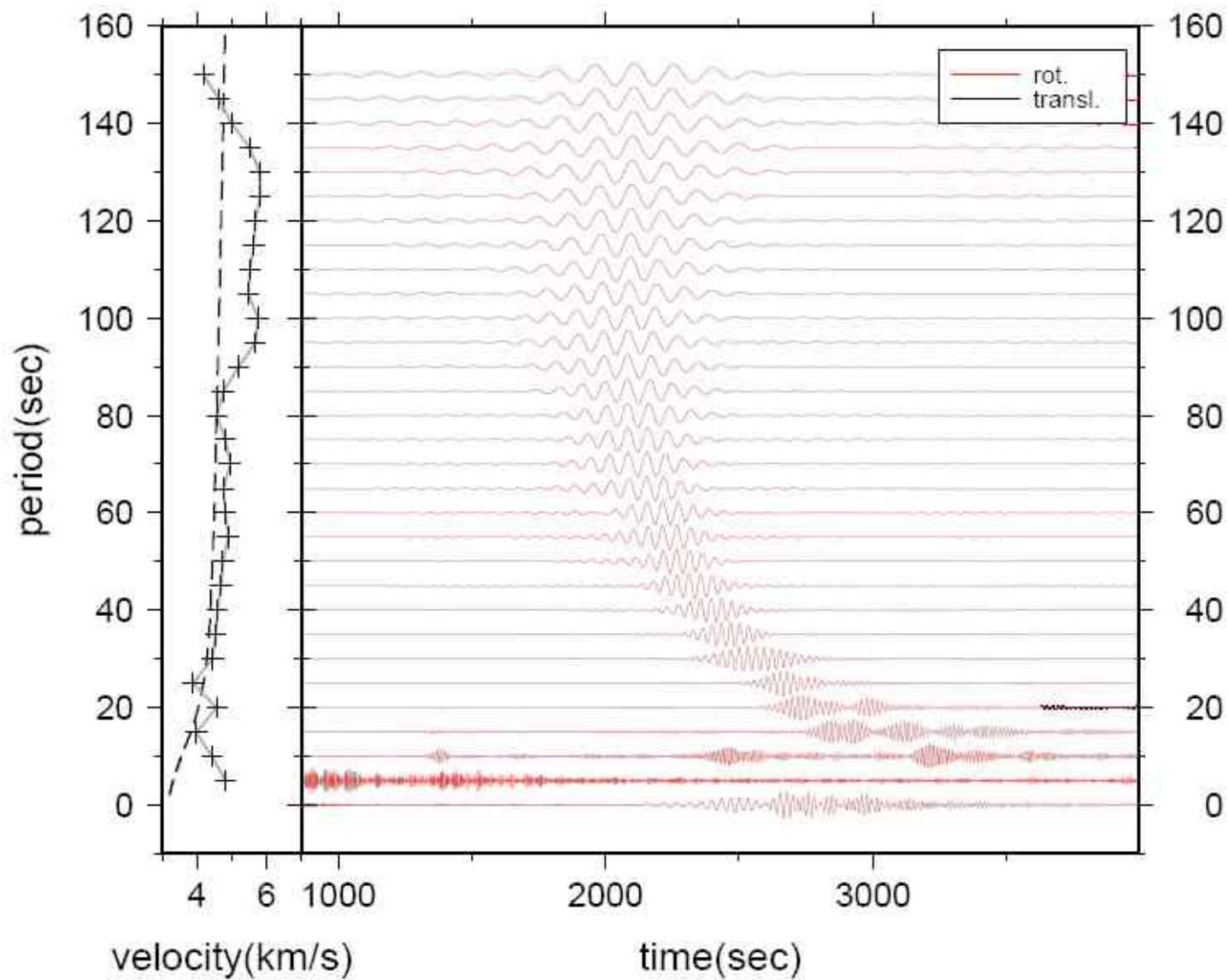
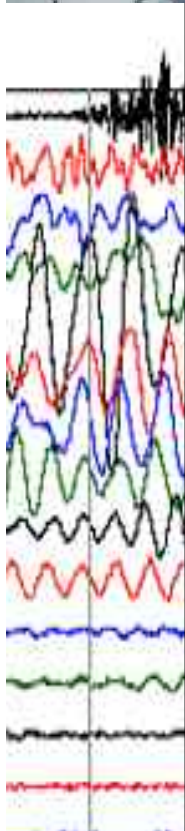


From Igel et al. (GJI, 2007)

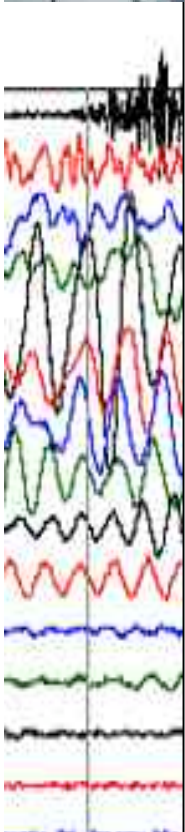


# ... dispersion ...

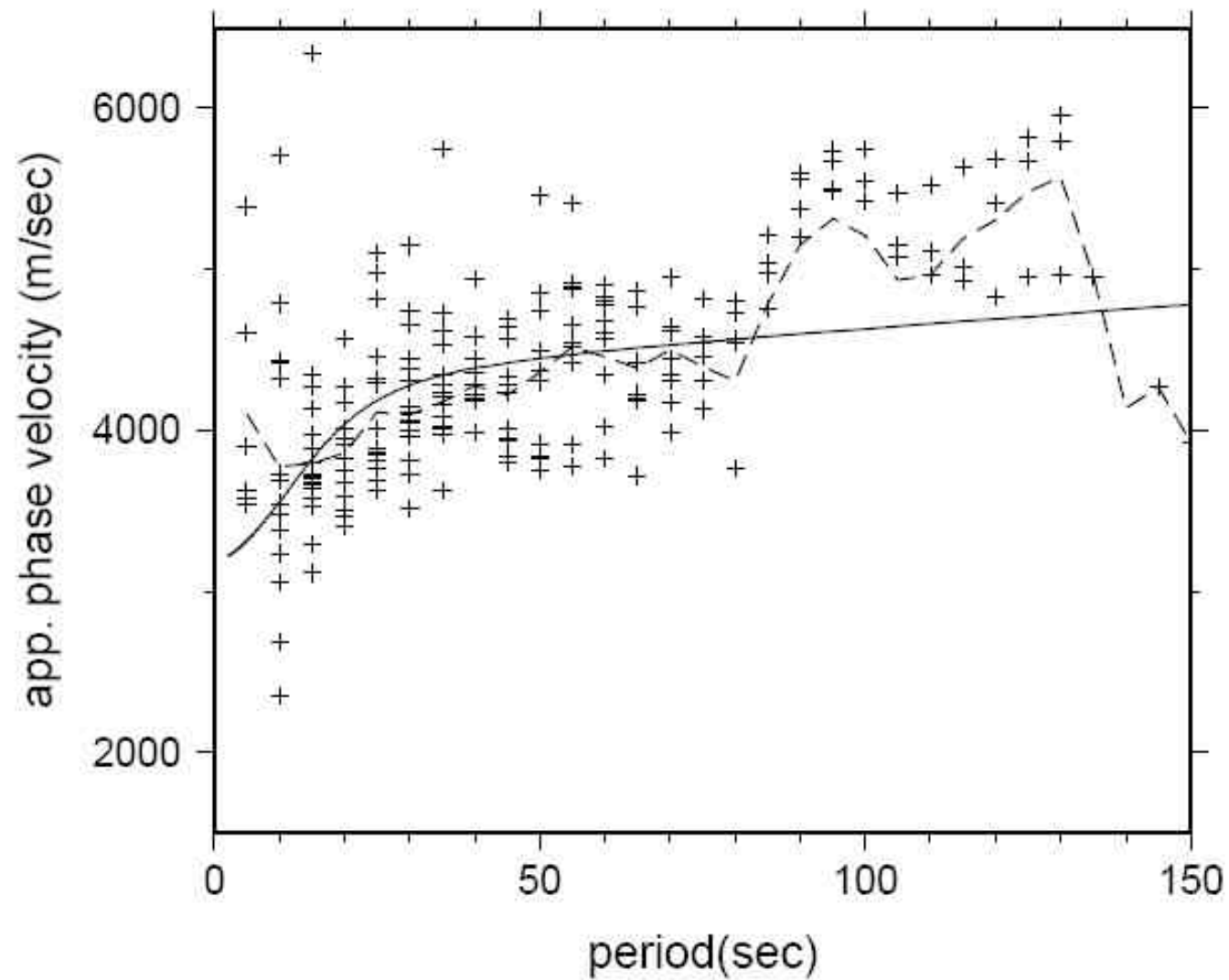
M7.4 Kuril islands, 15/1/09

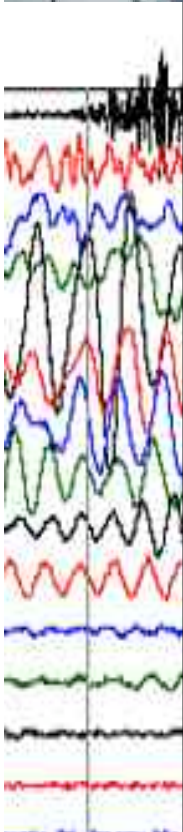




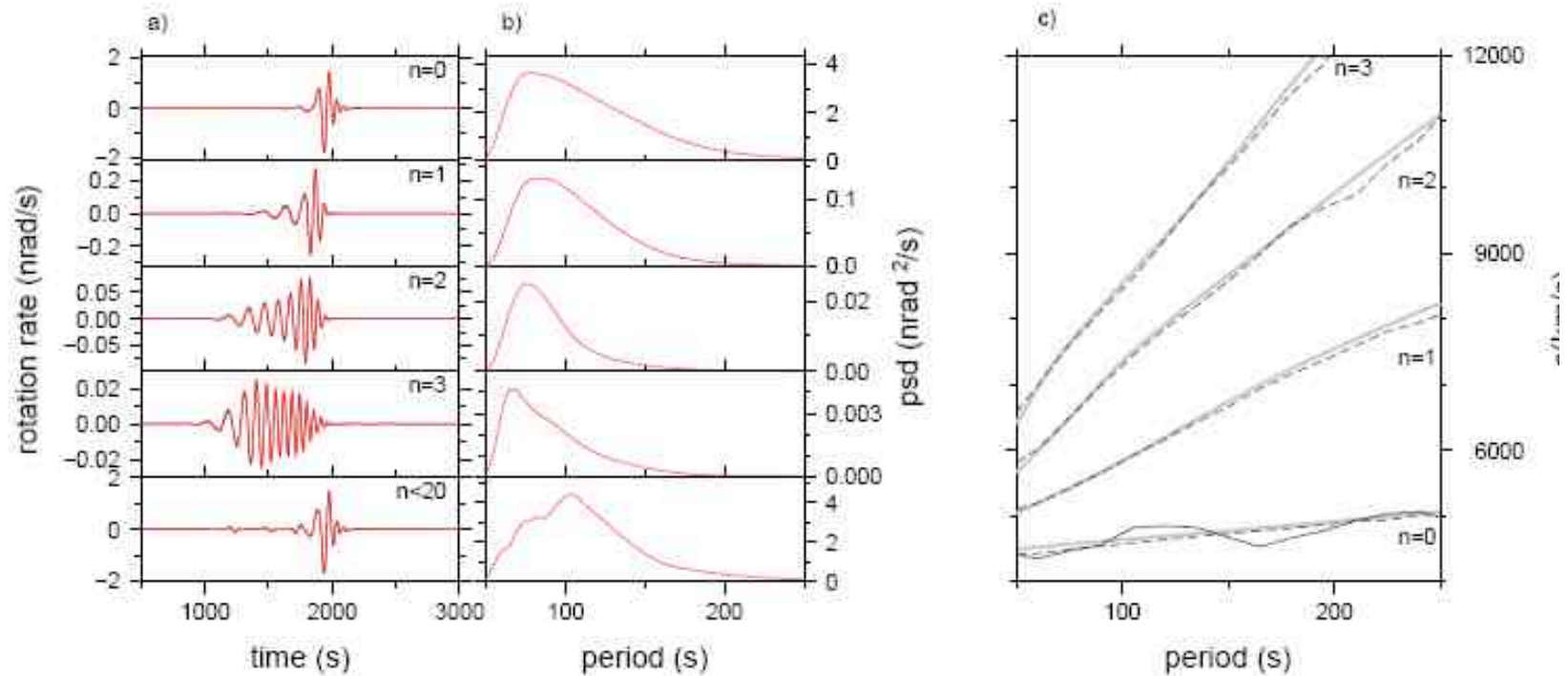


## Love wave dispersion from 28 events



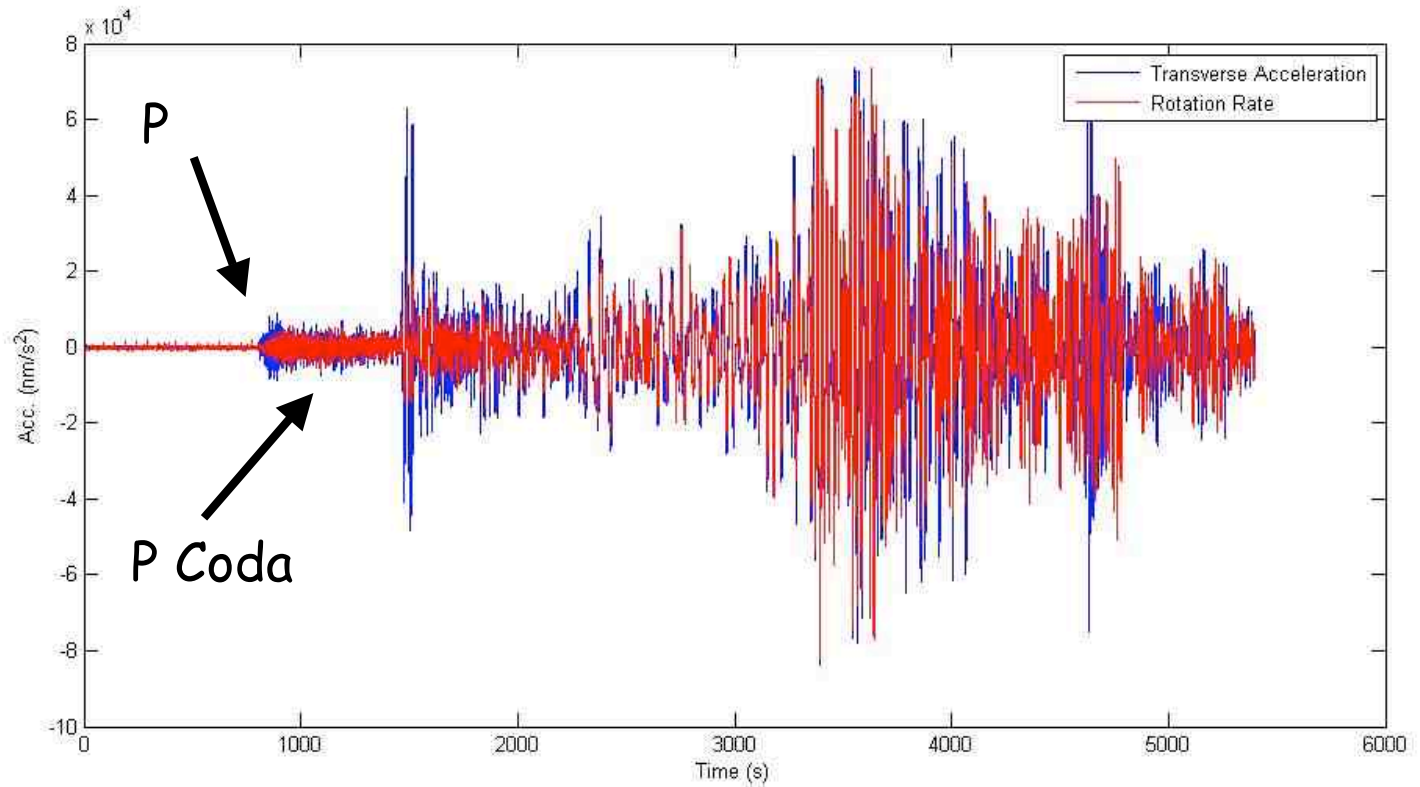
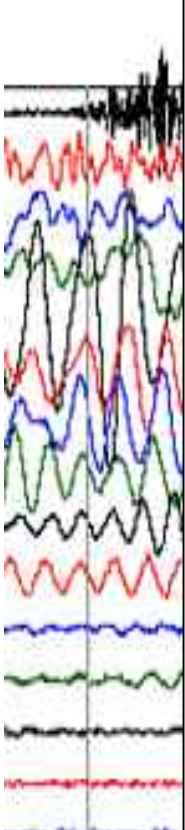


... why the large variations?



... overtones alone can do it ...  
(Kurrle et al., in preparation)

# Sumatra M8.3 12.9.2007



Modelling the ratio of Energy in P-Rotations and Translations allows putting constraints on crustal scattering (Pham et al., BSSA, 2009)

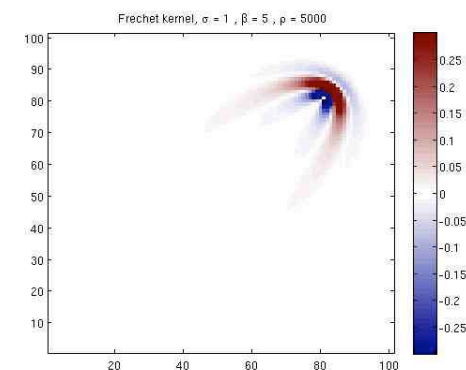
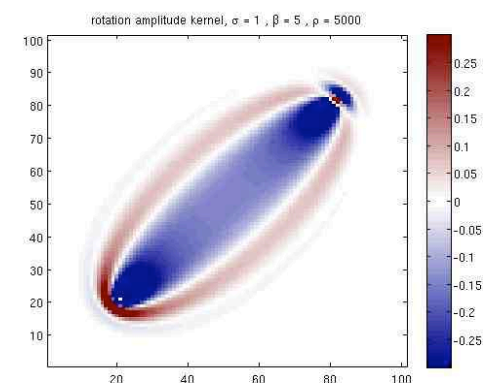
# Adjoint method and rotations *sensitivity kernels*

$$\beta_a(\mathbf{x}^r) := \frac{1}{2} \frac{A_v(\mathbf{x}^r)}{A_\omega(\mathbf{x}^r)},$$

$$A_v := \sqrt{\int_{\mathbb{R}} [F * (W\mathbf{v})]^2 dt}, \quad A_\omega := \sqrt{\int_{\mathbb{R}} [F * (W\omega)]^2 dt}$$

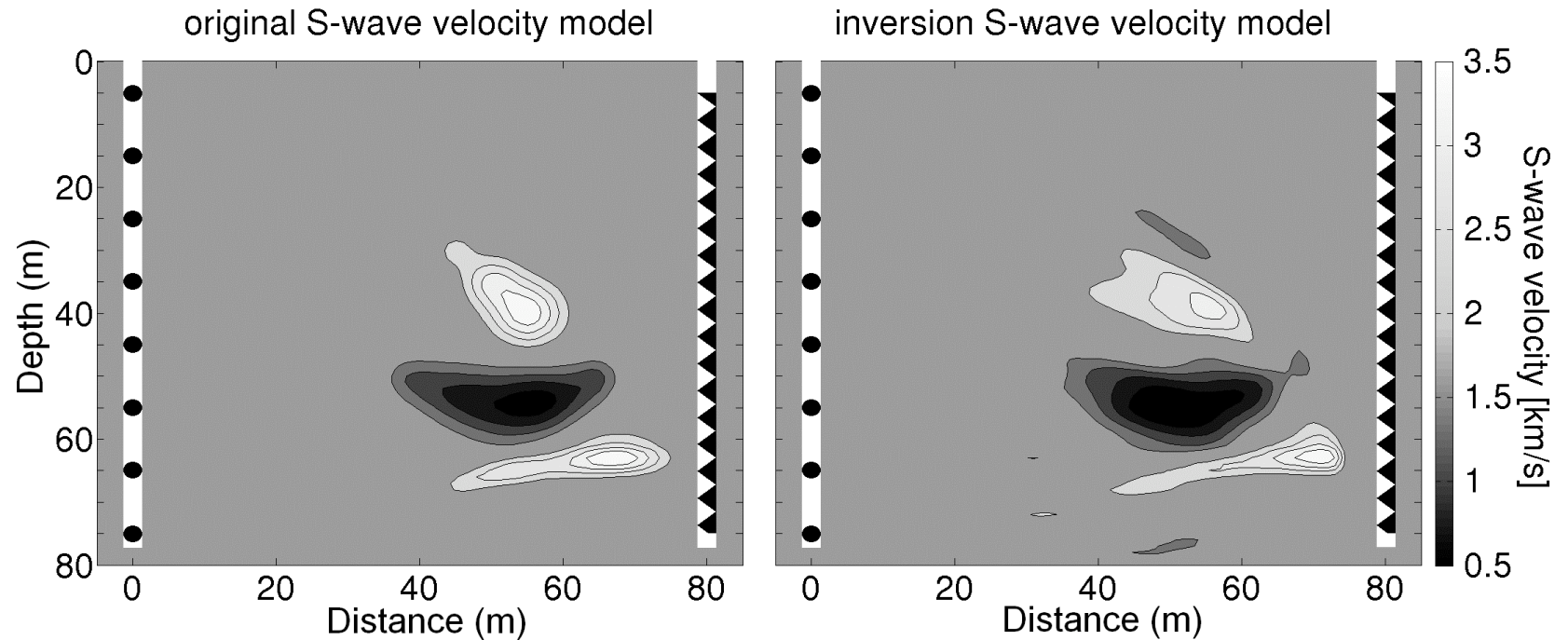
Sensitivity of travel times (or rotation amplitudes alone)

Sensitivity of acceleration/rotation  
(apparent shear wave speed)

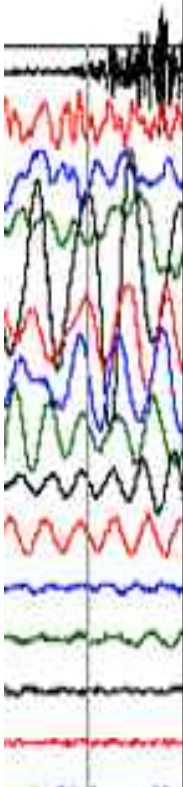




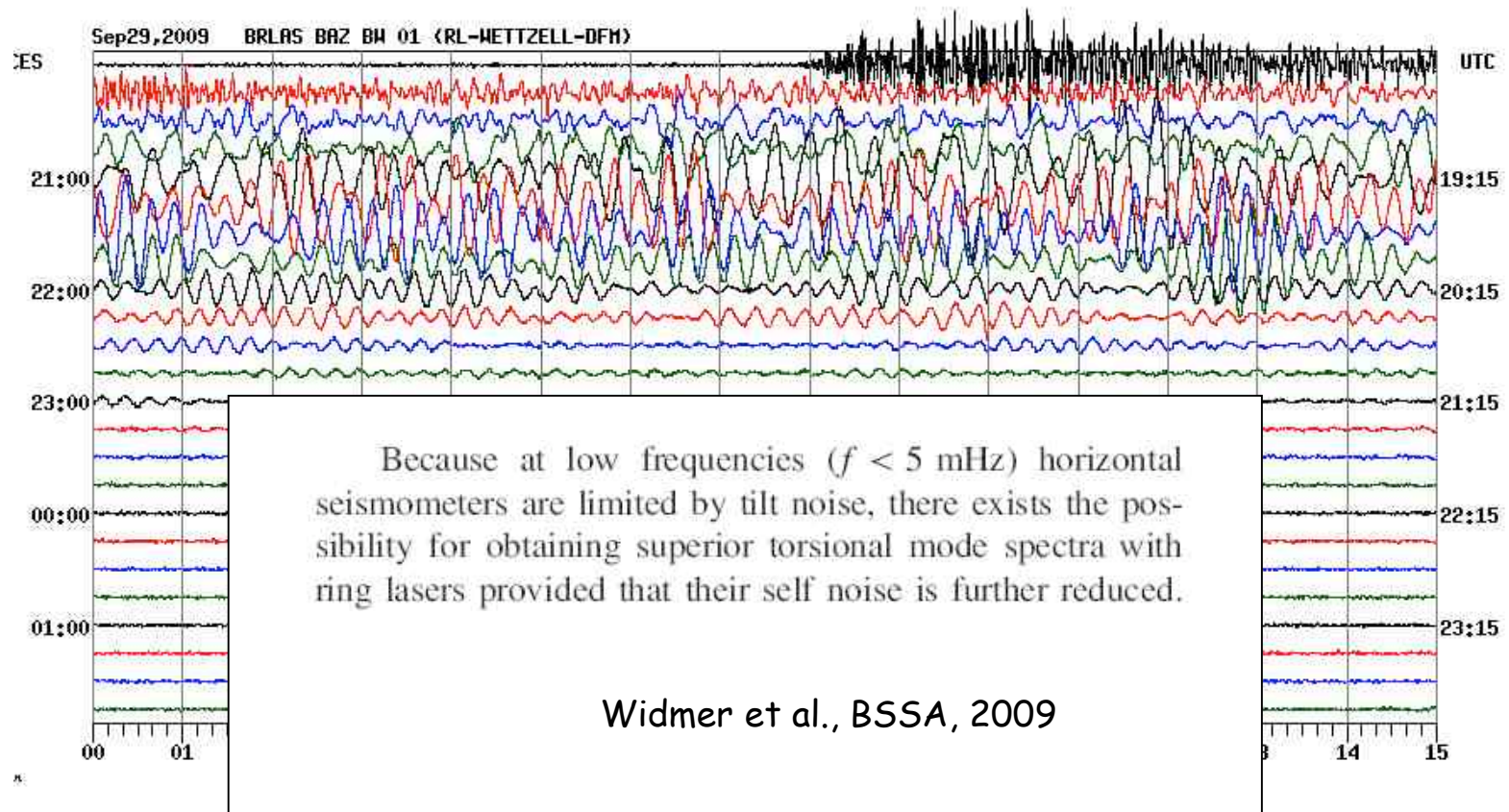
# ... tomography without travel times!



... the observables are frequency dependent amplitude ratios of rotations and translations ...



EARTHQUAKE on 29/09/2009 at 17:48  
(UTC) SAMOA ISLANDS REGION 164 km S  
Siumu MAGNITUDE: Mw 7.9

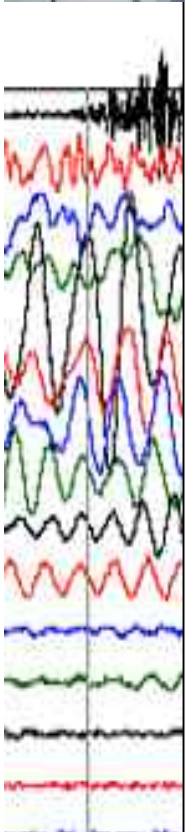
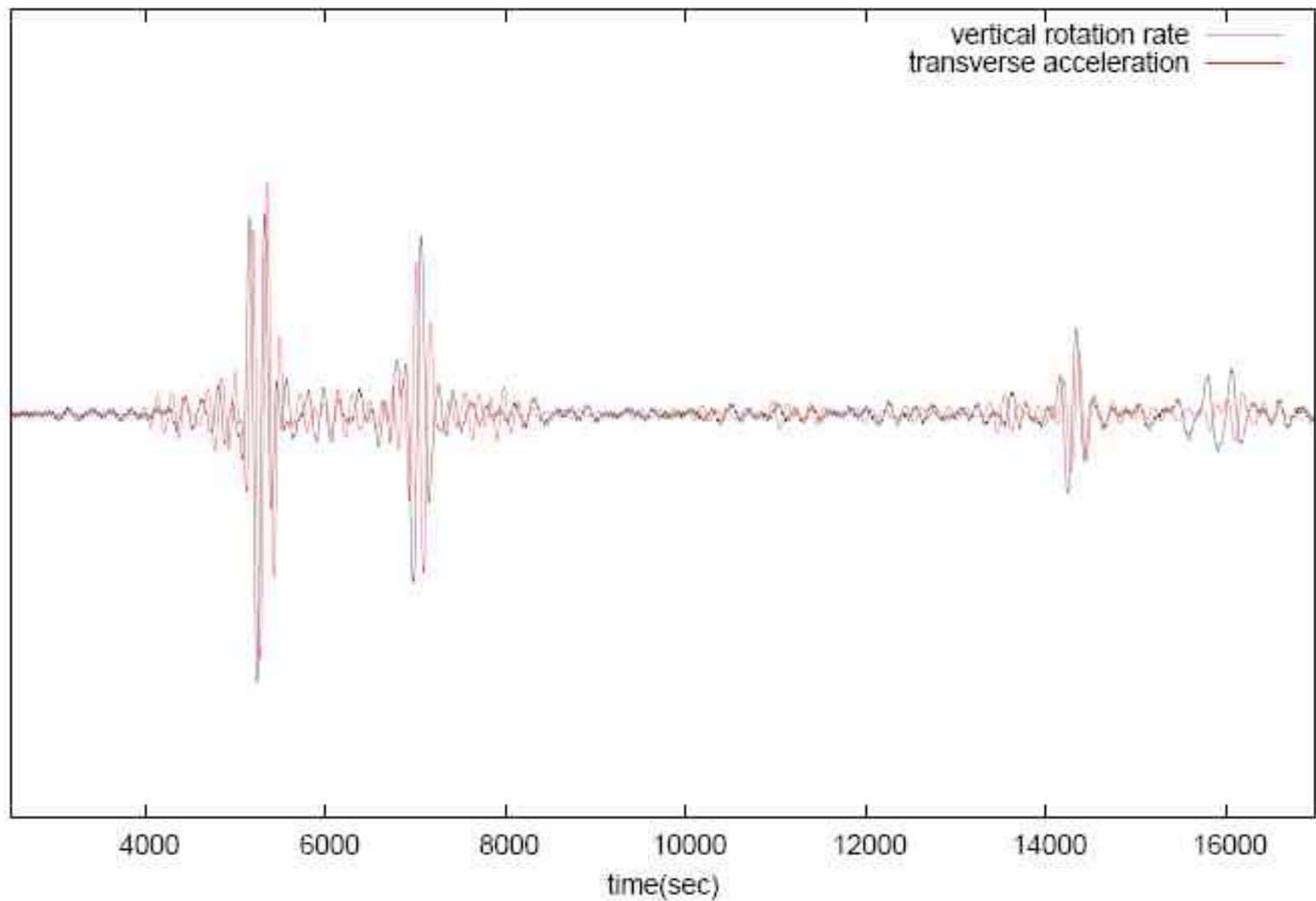


Ring laser record, Wettzell, Germany



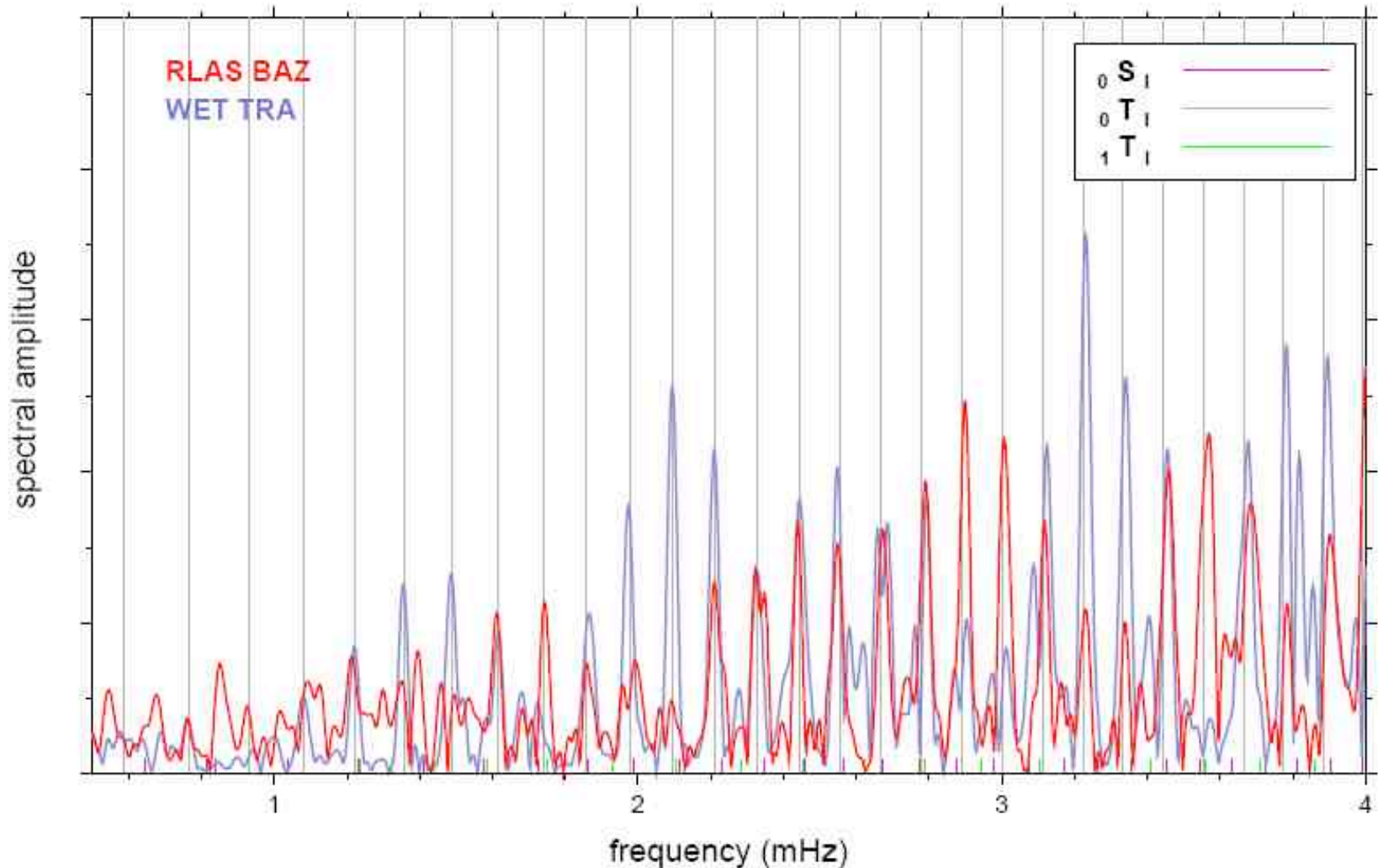
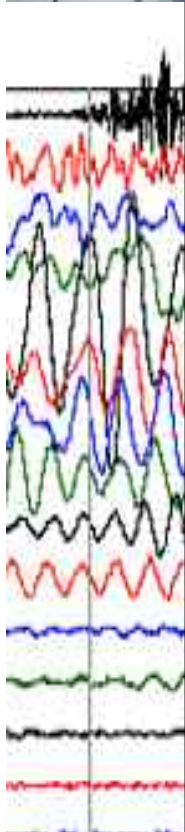
... long periods ....

real seismograms for Samoa earthquake



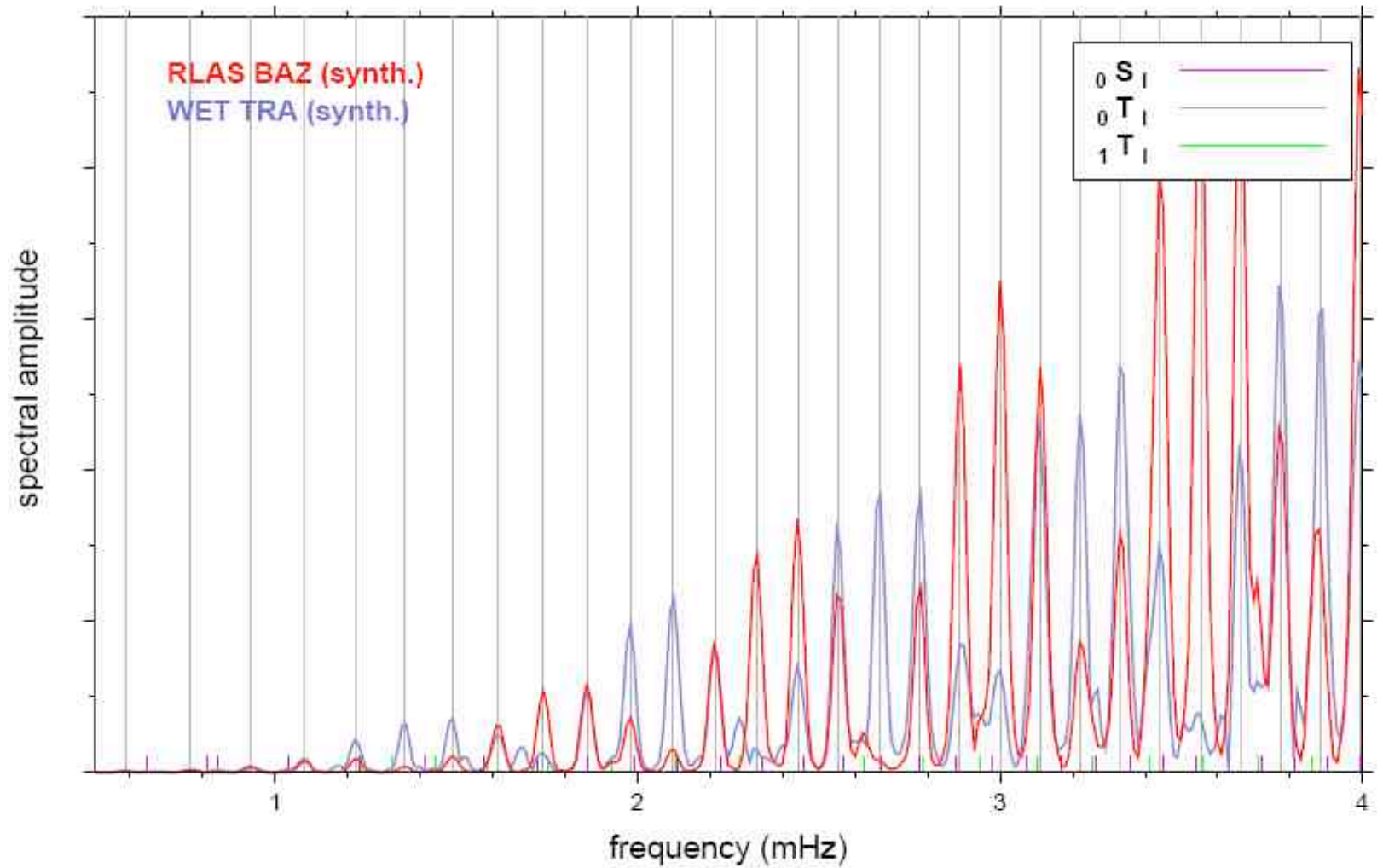
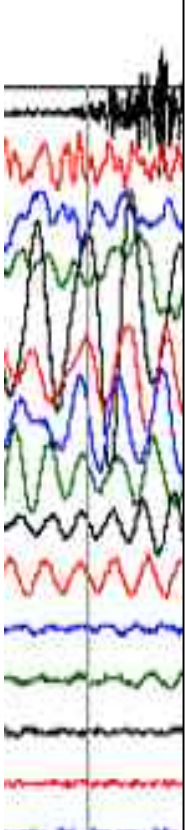
# First observations of eigenmodes with ring laser!

16 hours after 2009/09/29 Samoa earthquake



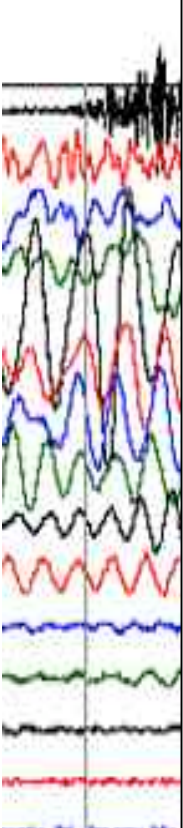
# Synthetics ....

16 hours after 209/09/29 Samoa earthquake





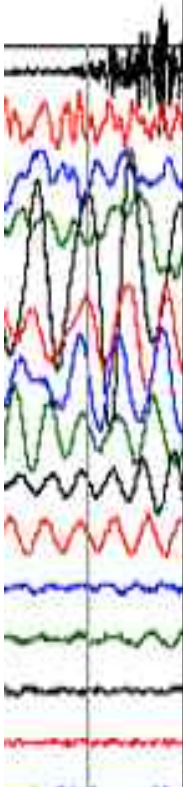
# Rotations using Seismic Arrays





# Array measurements

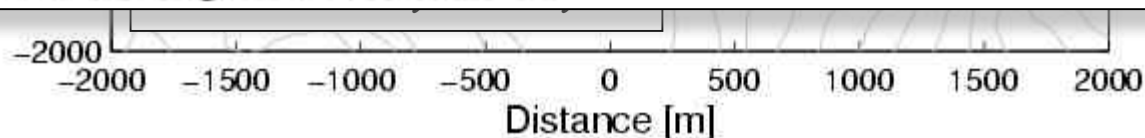
Dec 2003-Mar 2004

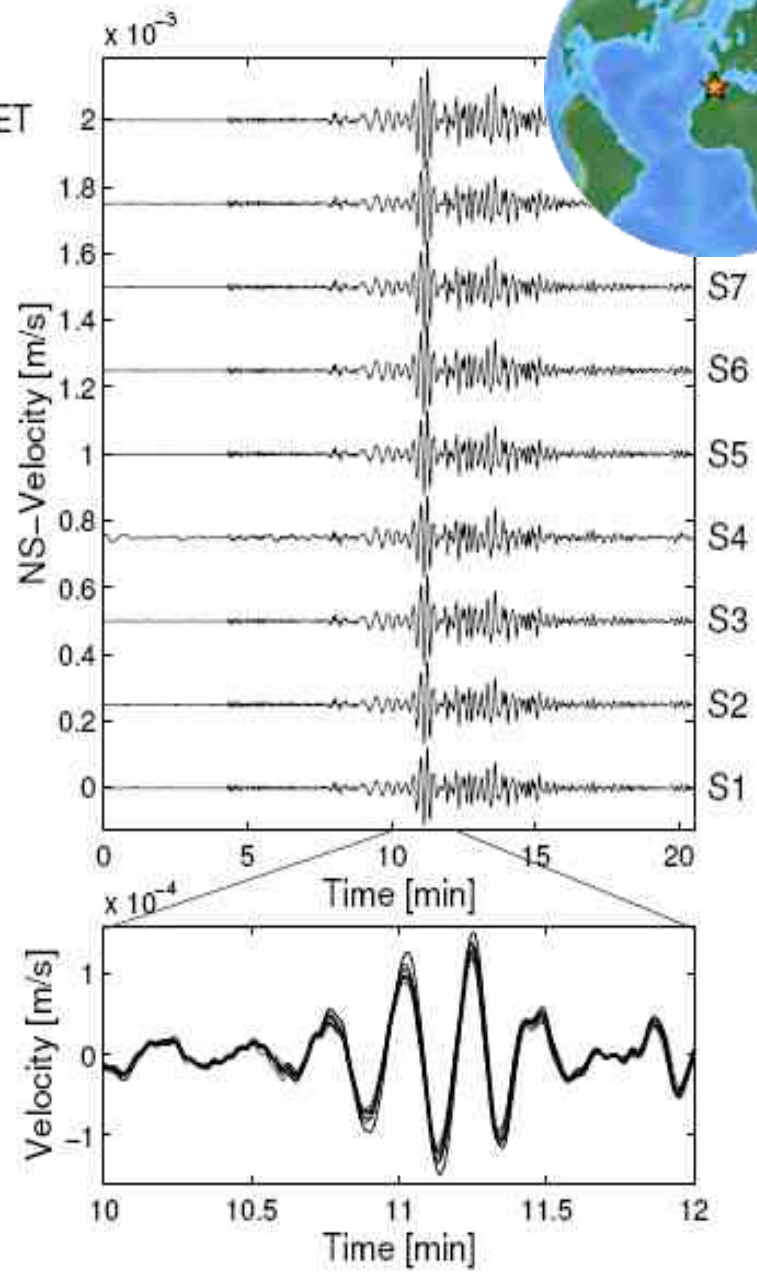
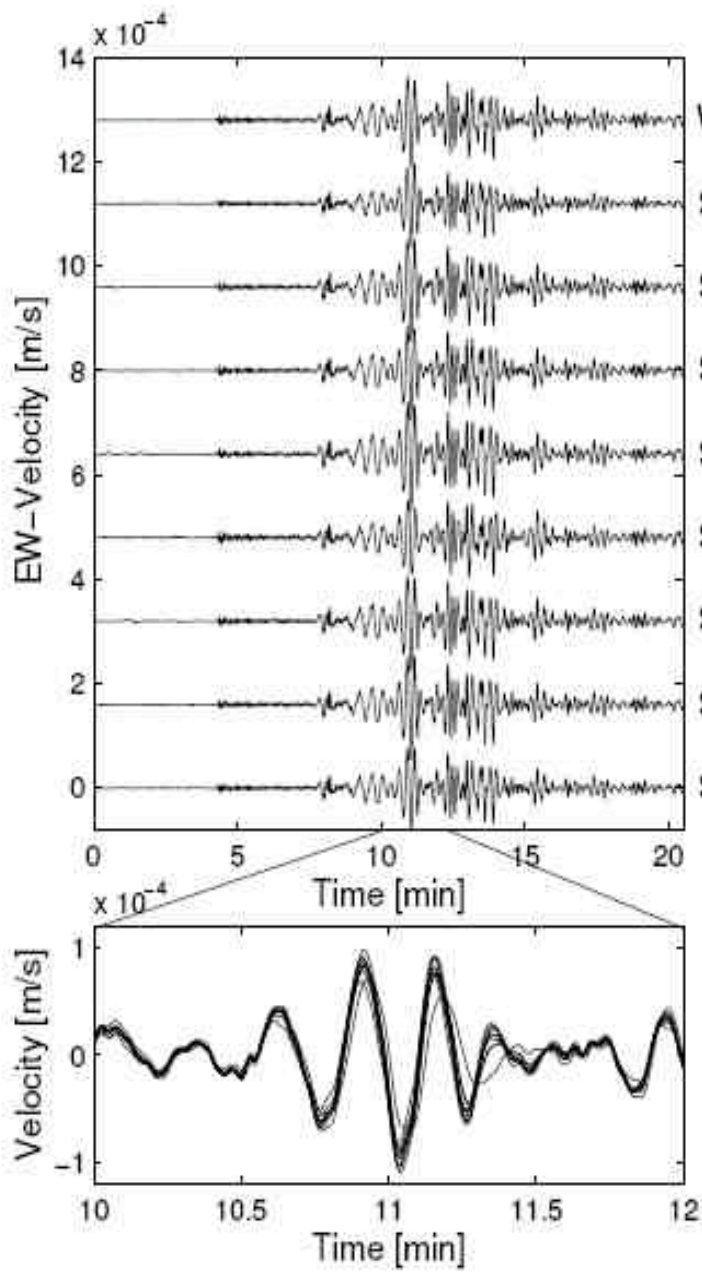
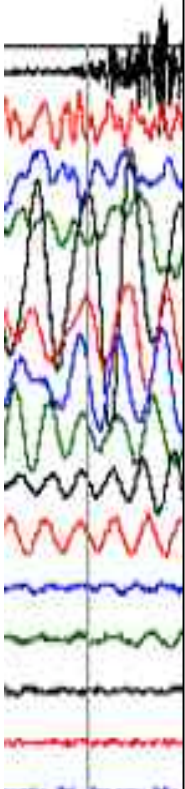


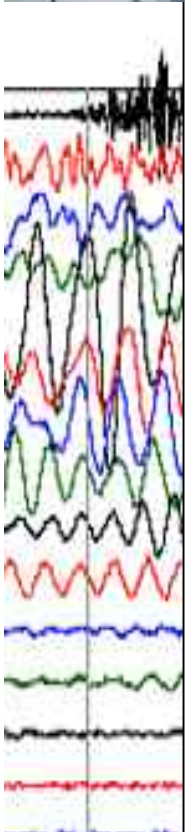
$$d_i = GR_i \quad (2)$$

$$= \begin{pmatrix} \partial_x u_x & \partial_y u_x & \partial_z u_x \\ \partial_x u_y & \partial_y u_y & \partial_z u_y \\ \partial_x u_z & -\partial_z u_y & -\eta(\partial_x u_x + \partial_y u_y) \end{pmatrix} R_i,$$

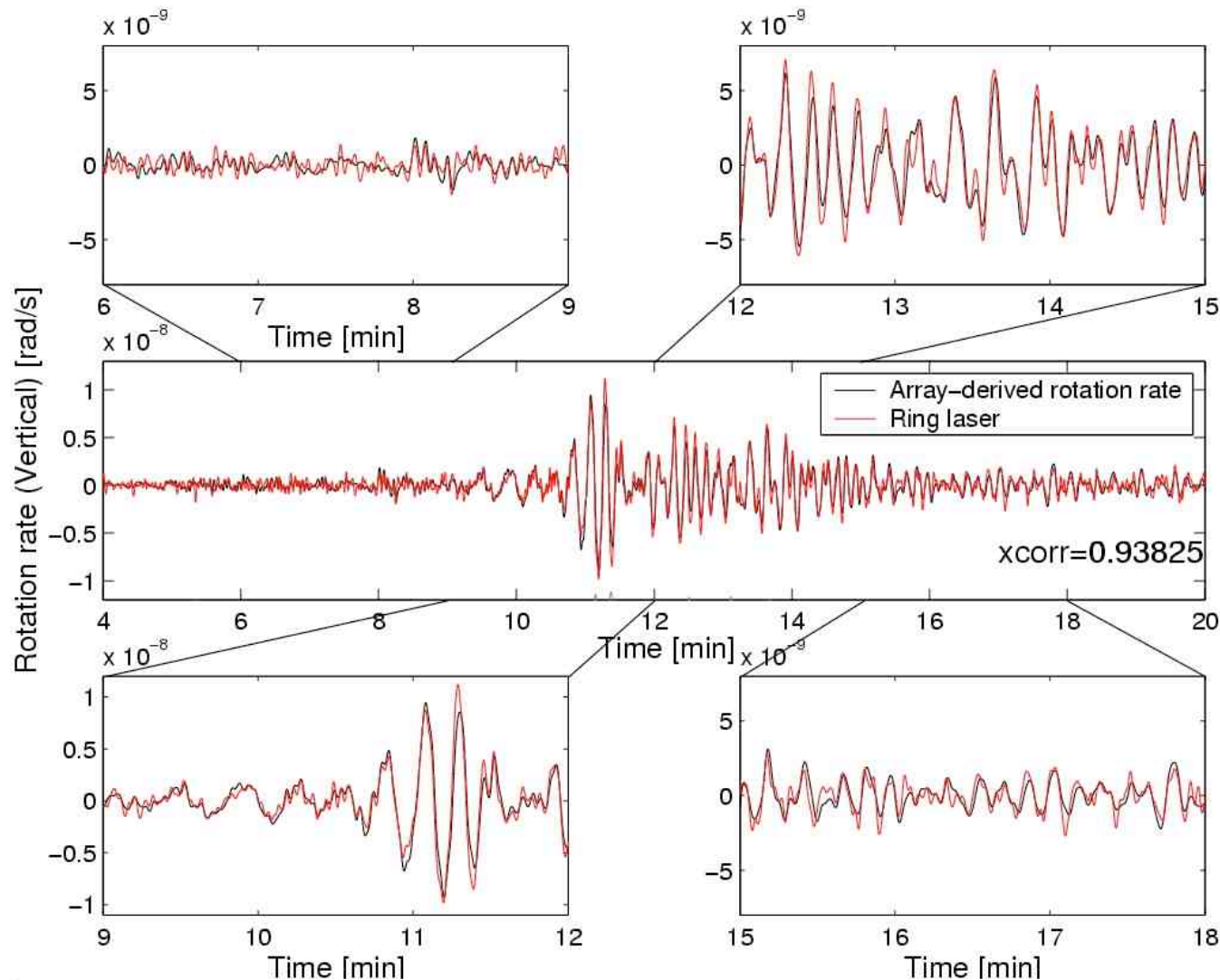
where,  $\eta = \lambda(\lambda + 2\mu)$ ,  $\lambda$  and  $\mu$  are the Lamé parameters,  $d_i = u_i - u_0$ ,  $R_i = r_i - r_0$ ,  $u_i$ ,  $r_i$ , and  $u_0$ ,  $r_0$  are the displacements at the coordinates of the  $i$ th station and the reference station (subscript 0), respectively. At least three stations must be used to determine the horizontal-displacement gradient using this method.







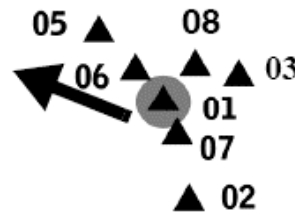
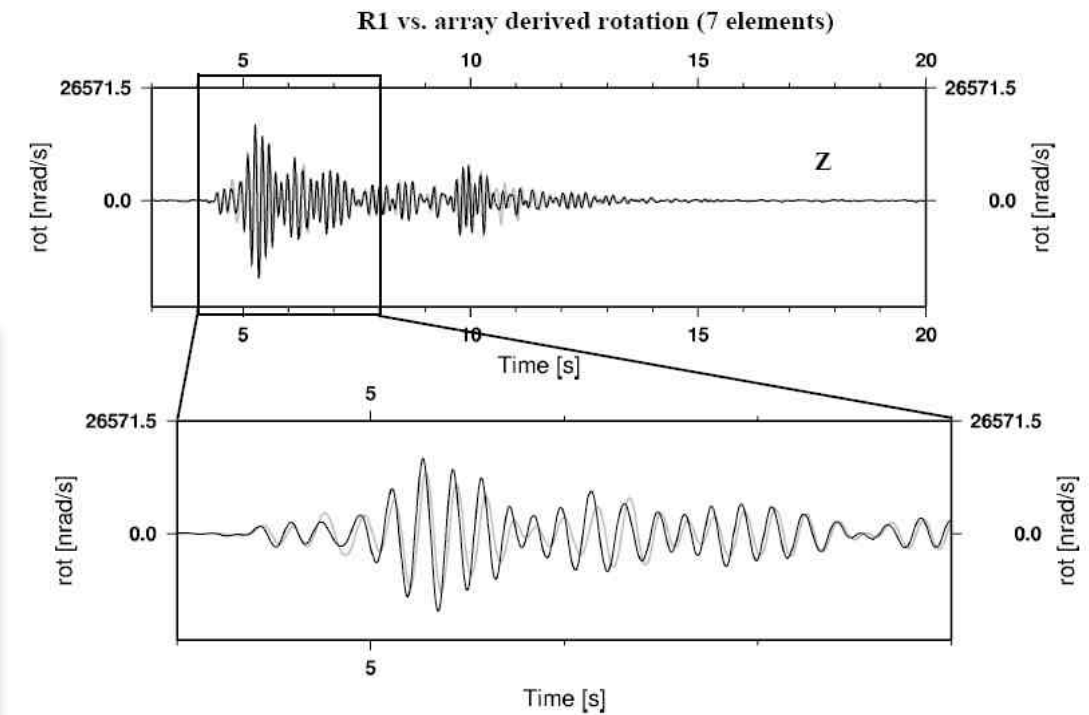
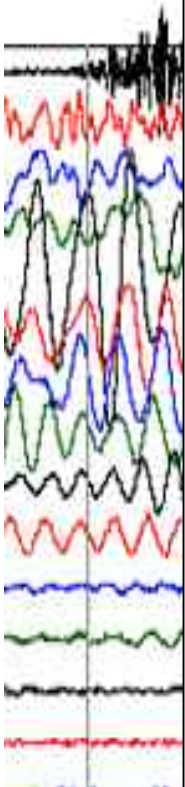
**First** comparison of array-derived rotations (black) and direct ring laser measurements (**red**)



From Suryanto et al (2006, BSSA)



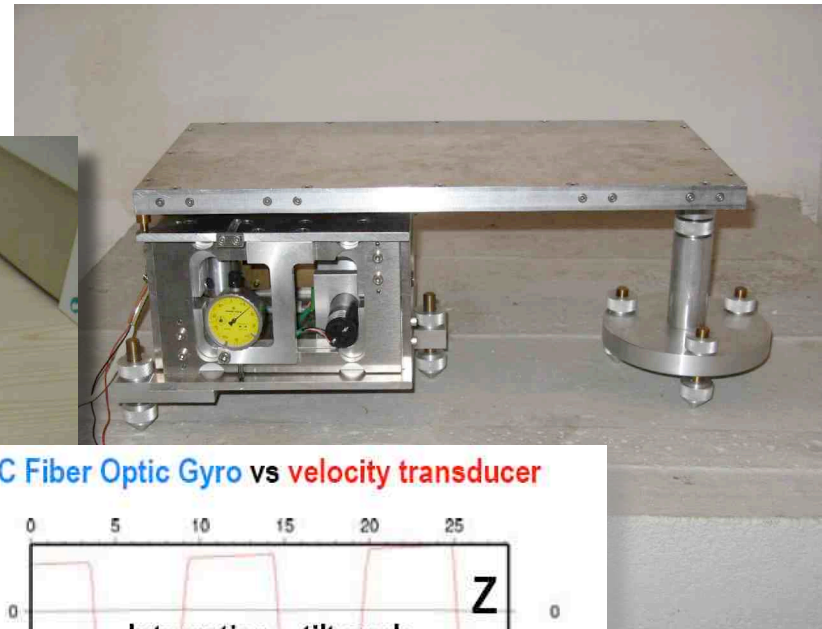
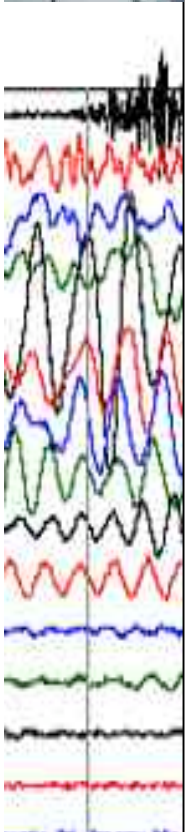
# Array vs. direct



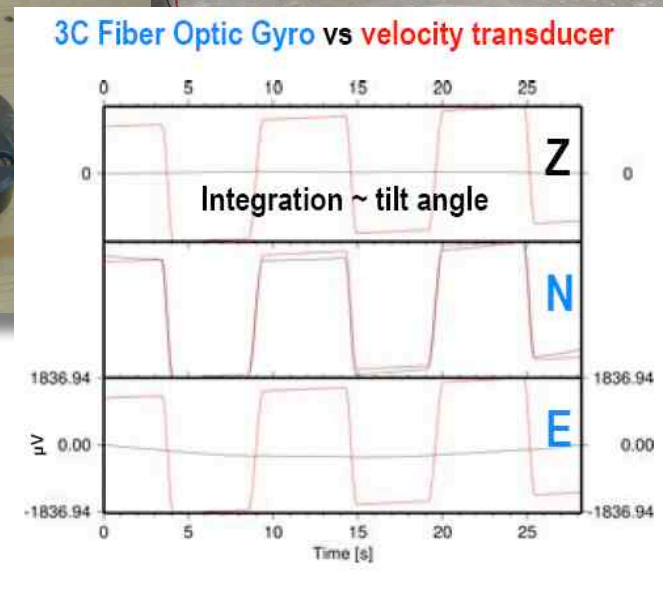
Wassermann et al., 2009, BSSA.



# Step table tests with low-resolution sensors

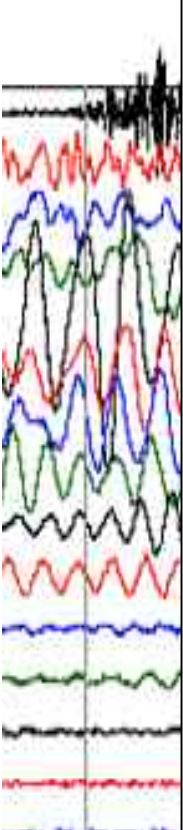


See poster by  
Wassermann et al!



# Conclusions

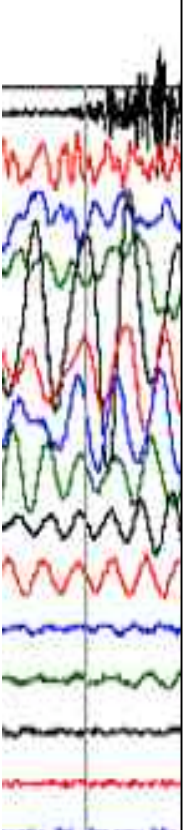
- Ring laser technology is the most promising technology for broadband far-field observations (Hz - mHz range,  $10^{-11}$ - $10^{-6}$  rad/s)
- Measuring tilts (horizontal component of rotation) in broadband seismology is a fundamental research question!
- Collocated observations of rotations and translations are providing additional structural information!
- Our current state of knowledge:
  - Broadband, far field: ring lasers
  - Seismological applications, portable: fiber-optic gyros
  - Strong motion: fluid-based sensors (e.g., R1)



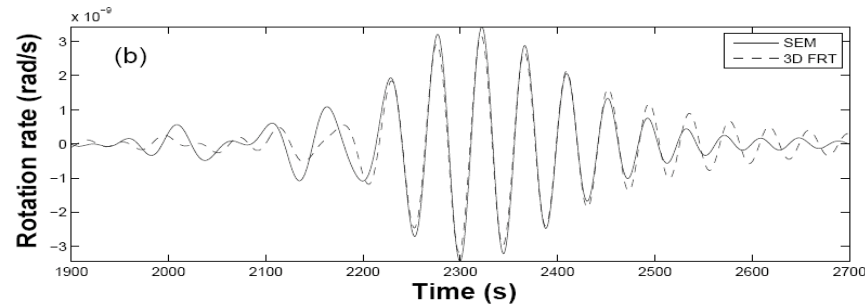
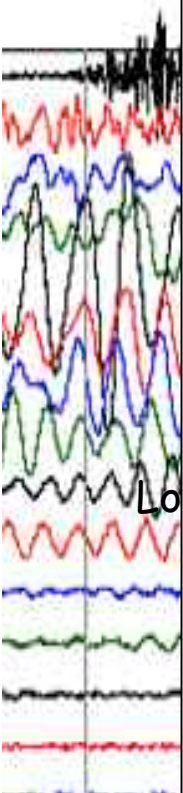
# Future

- A 6C permanent observatory (with surrounding BB array) at PFO
- Cooperation with Gravitational Wave Experiments (e.g., LIGO, VIRGO)
- Applications in earthquake engineering, source problems, reservoir seismics, strong motion seismology, ocean bottom seismology, planetology
- Tilts, tilts, tilts!

- > [www.rotational-seismology.org](http://www.rotational-seismology.org) (IWGoRS)
- > IWGoRS Meeting in Europe, provisional schedule for 2nd Meeting in Prague 12-15 October, 2010



# Using Surface Wave Ray Theory



$$\dot{\Omega}_r = \omega \frac{(l + 1/2)}{2a} W Y_l^m = \omega \frac{k}{2} W Y_l^m$$

$$\ddot{s}_T = \omega^2 W Y_l^m$$

$$l \gg 1$$

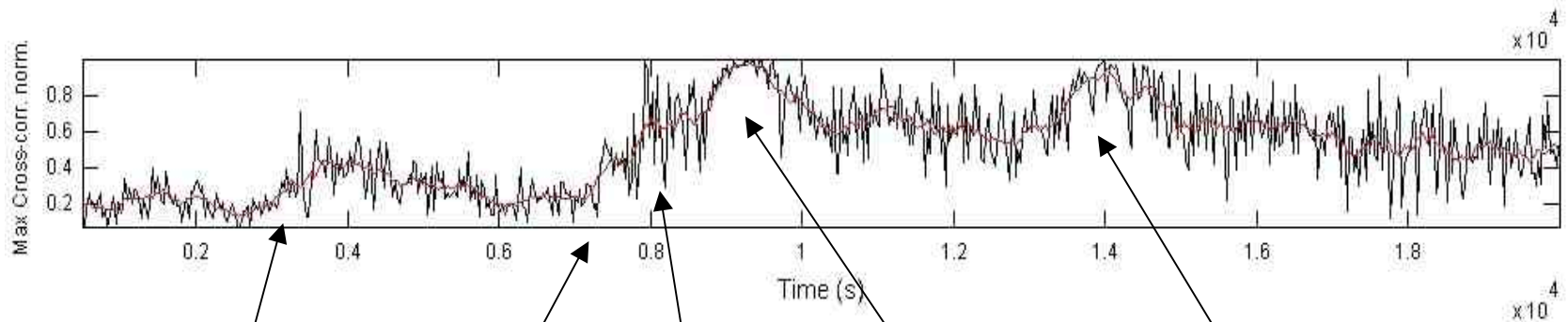
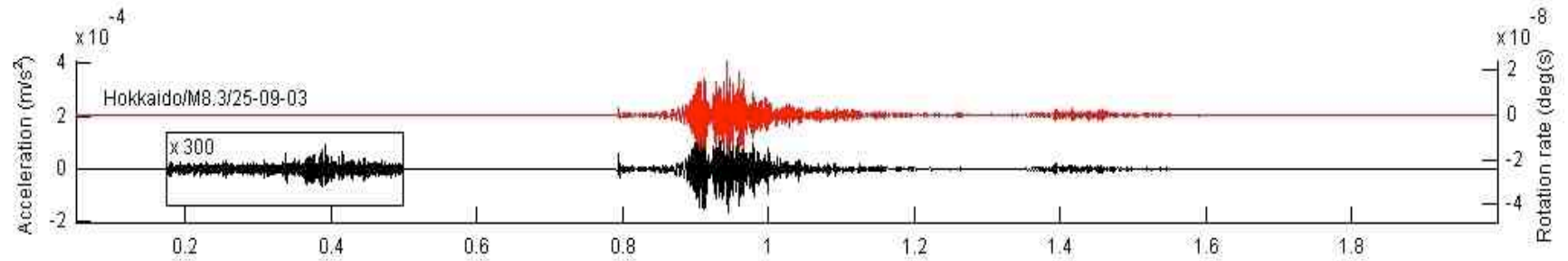
Love wave dispersion from **point measurement!**

$$\frac{\dot{\Omega}_r(\omega)}{\ddot{s}_T(\omega)} = \frac{k}{2\omega} = \frac{1}{2c_T(\omega)}$$





# Max. cross-corr. coefficient in sliding time window transverse acceleration - **rotation rate**



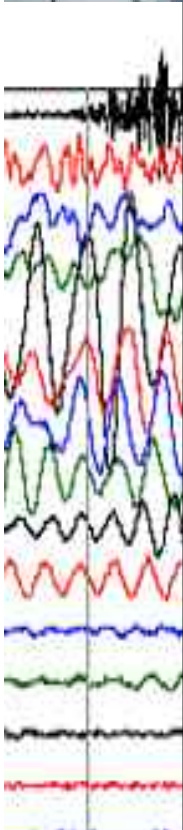
P-onset

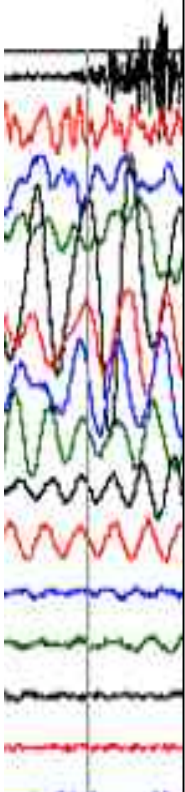
S-wave

Love waves

Aftershock

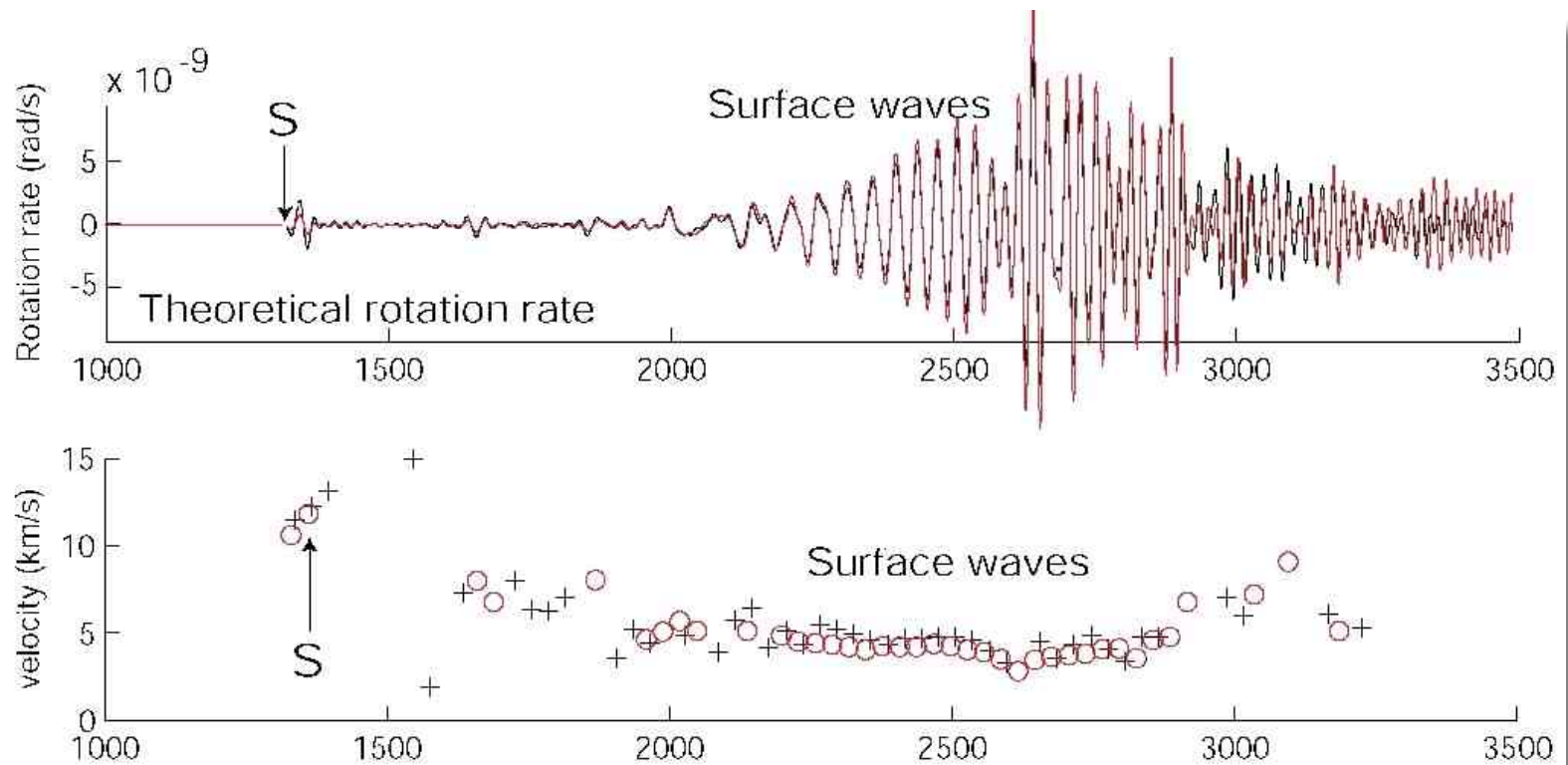
Small tele-seismic event





# M8.3 Tokachi-oki, 25 September 2003

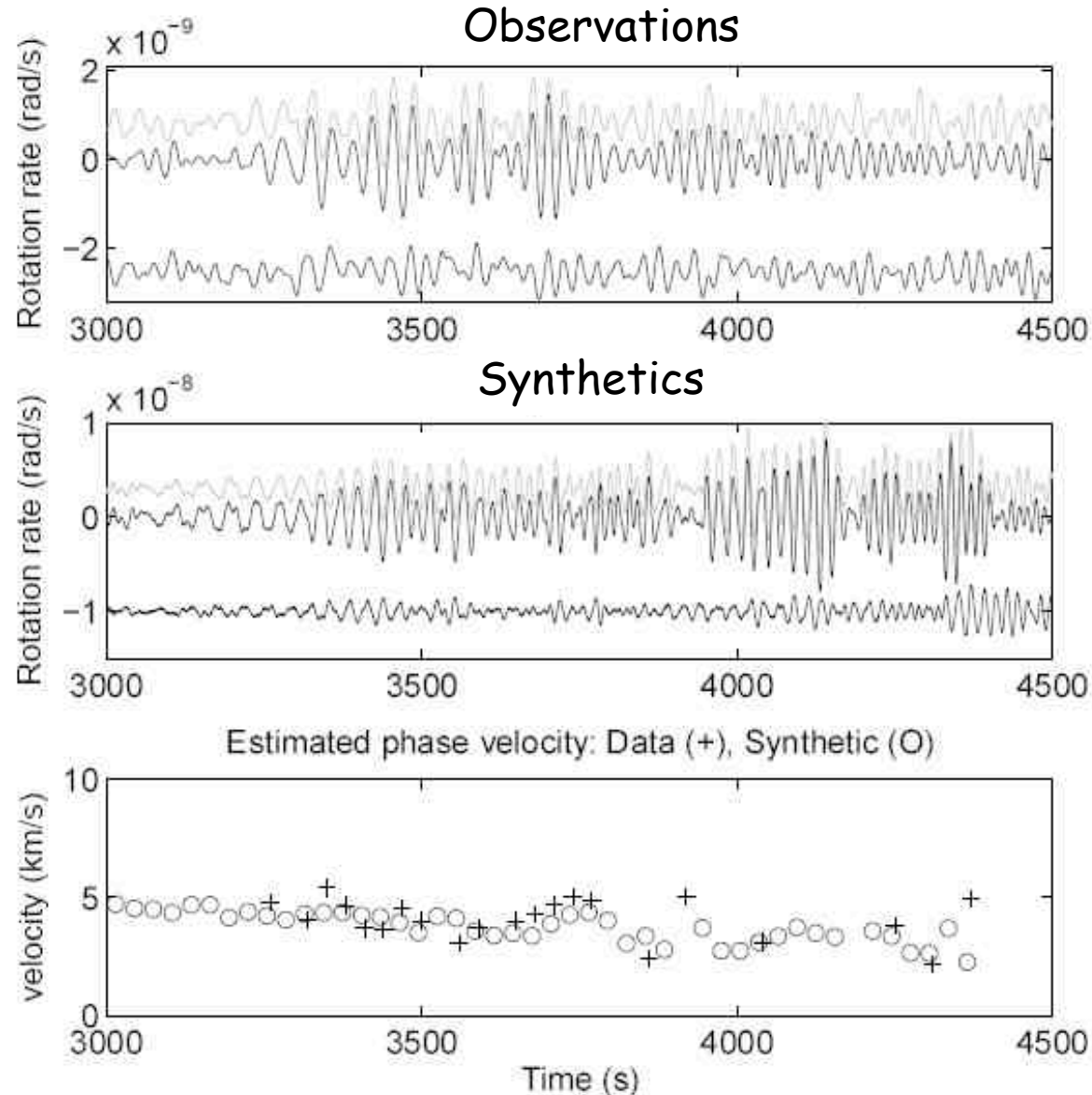
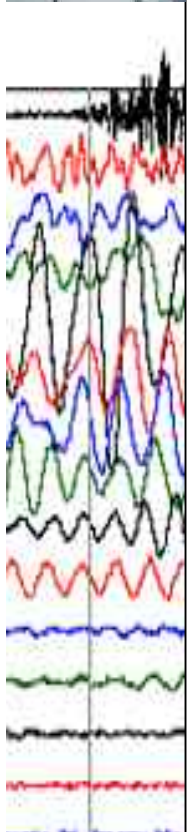
phase velocities ( + observations, o theory)



Horizontal phase velocity in sliding time window

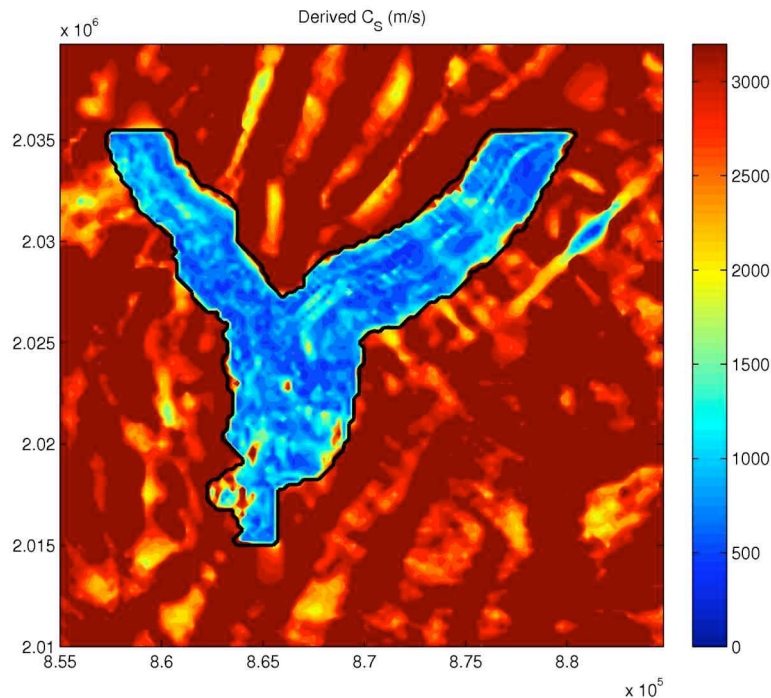
From Igel et al. (GRL, 2005)

# Real vs. Synthetics : Papua event

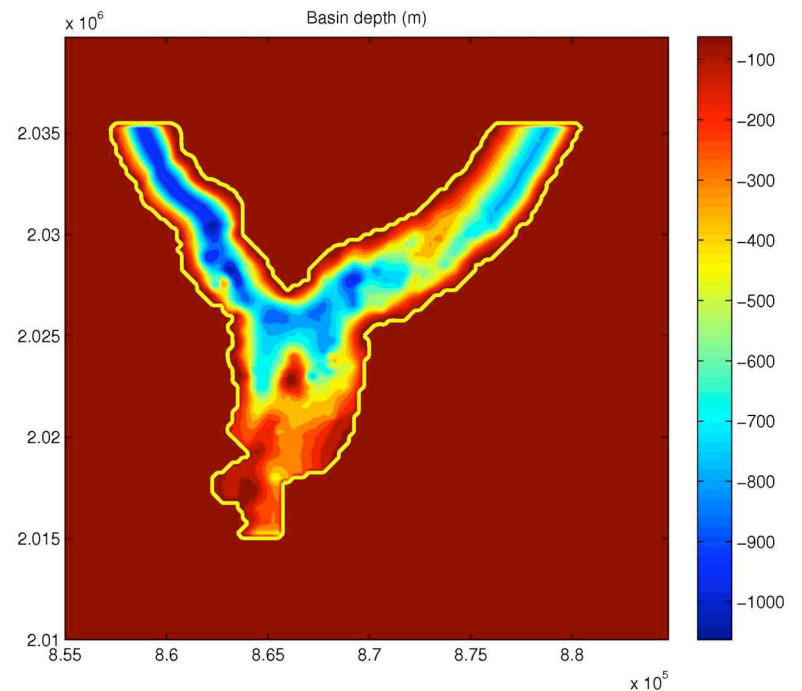


Cochard et al., 2006

# Collocated measurements of translations and rotations Grenoble Basin (synthetics) and the LA basin



Apparent shear velocity from  
ratio acceleration/rotation

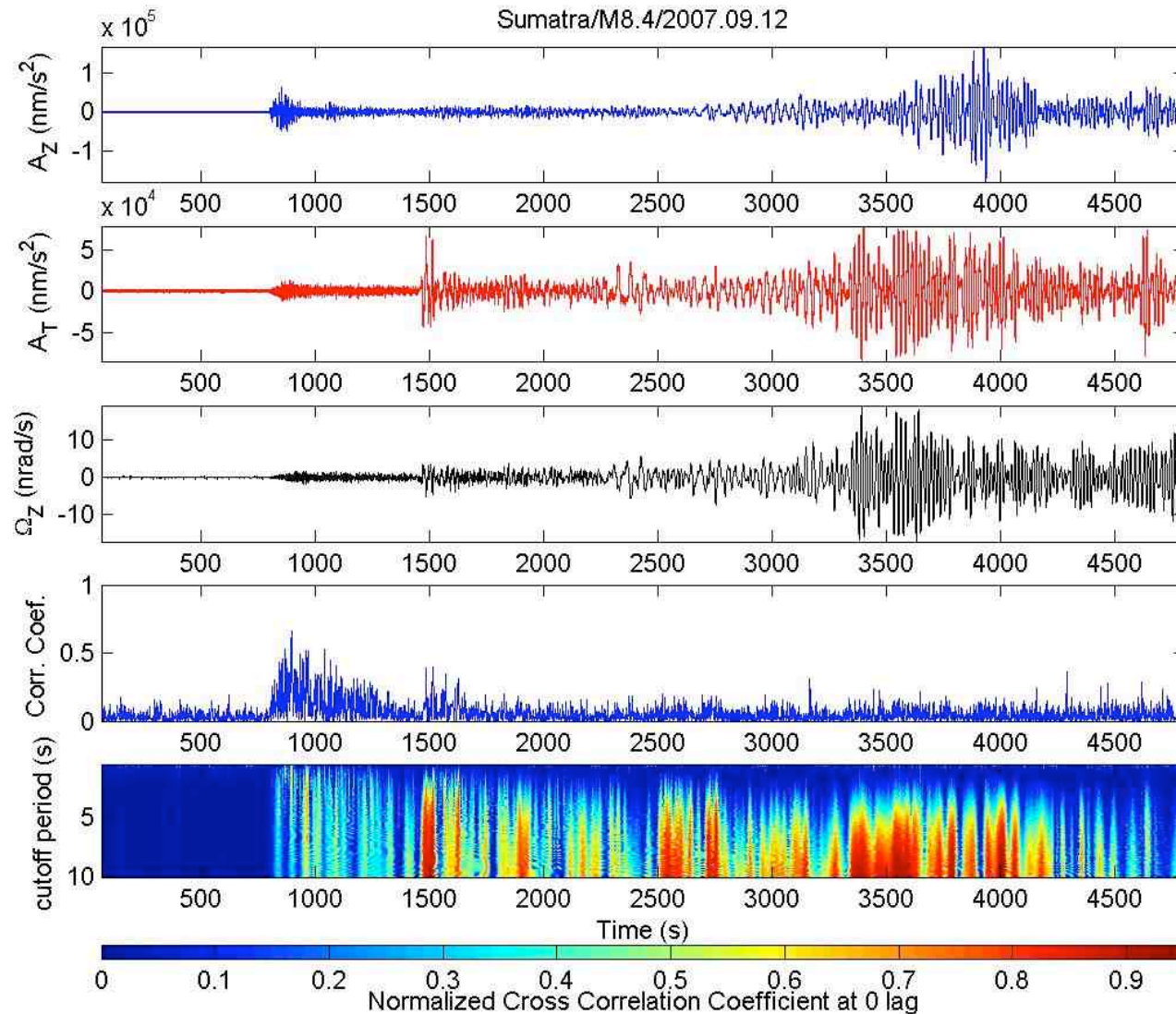
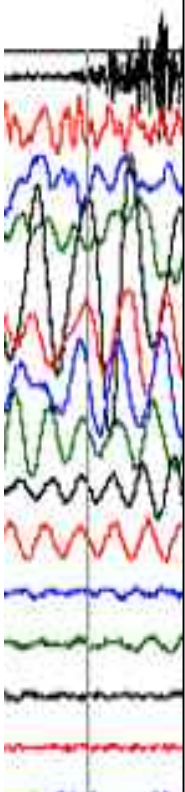


Basin model

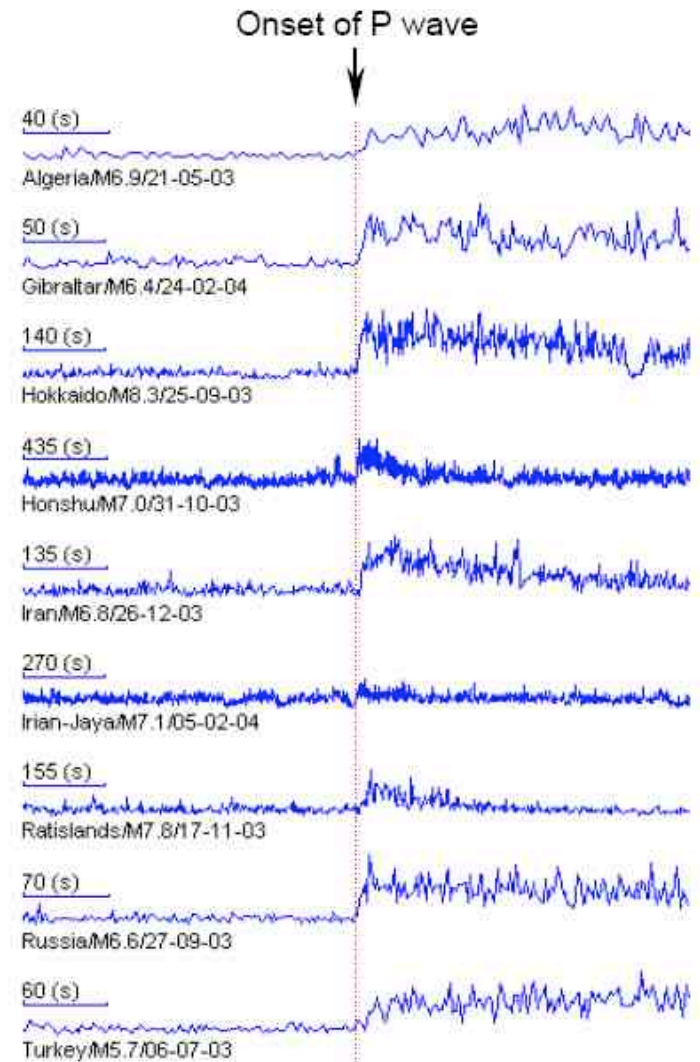
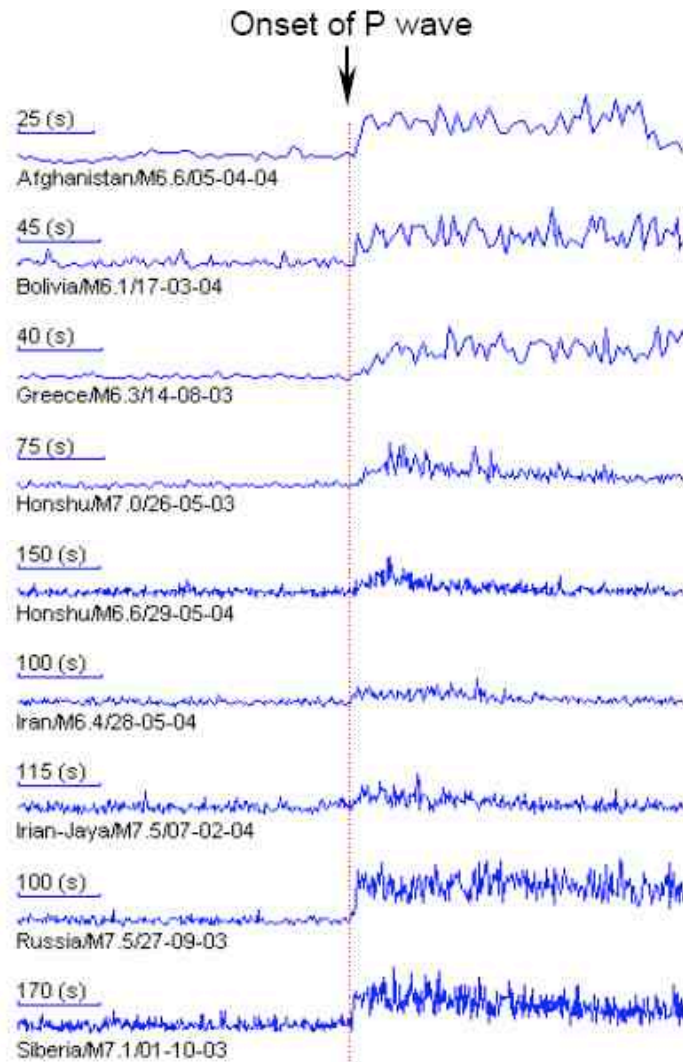
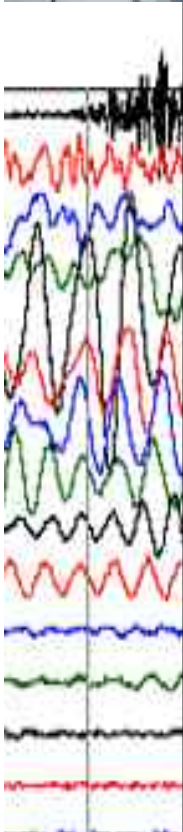
Stupazzini et al., 2009, BSSA, and Wang et al. BSSA, 2009



# Rotational signals in the P-coda? *frequency dependence*

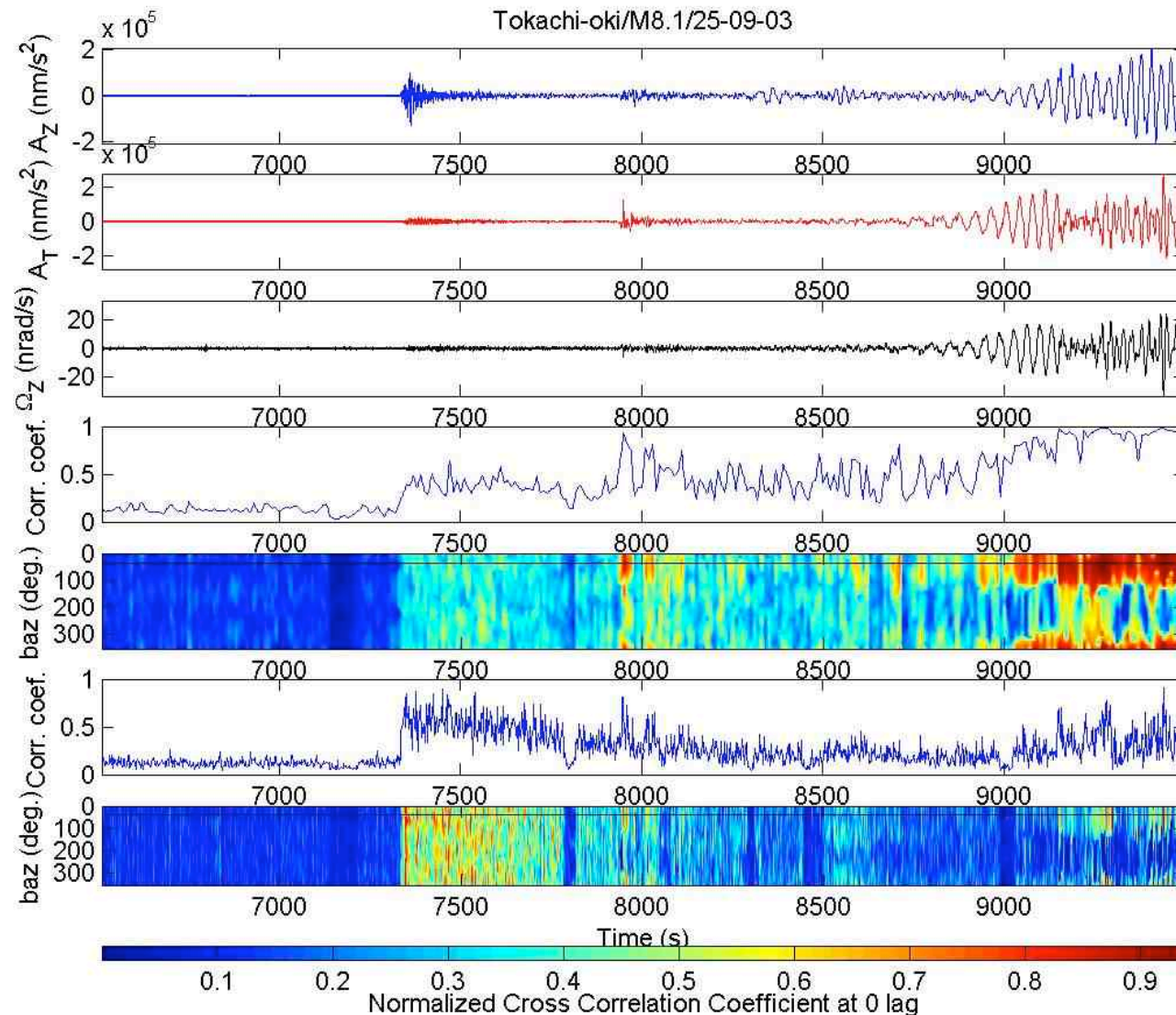
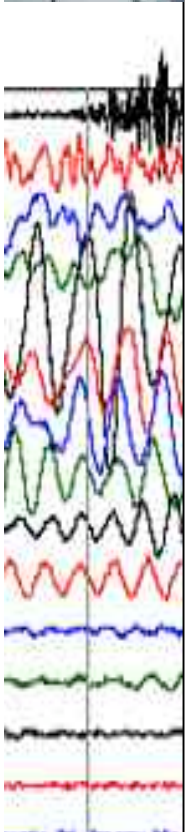


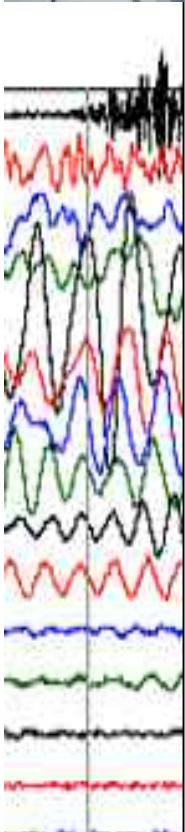
... observable for all events!





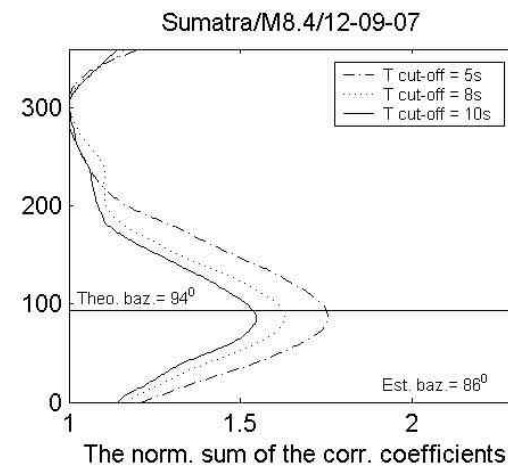
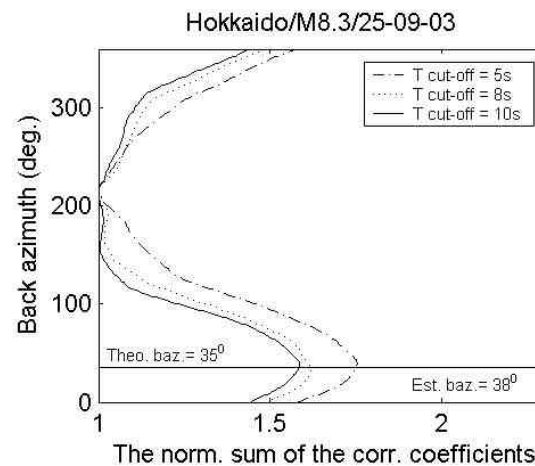
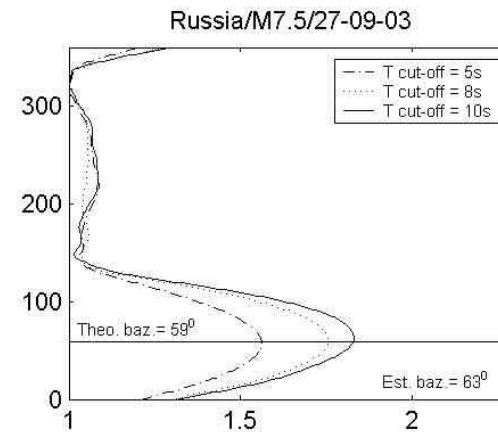
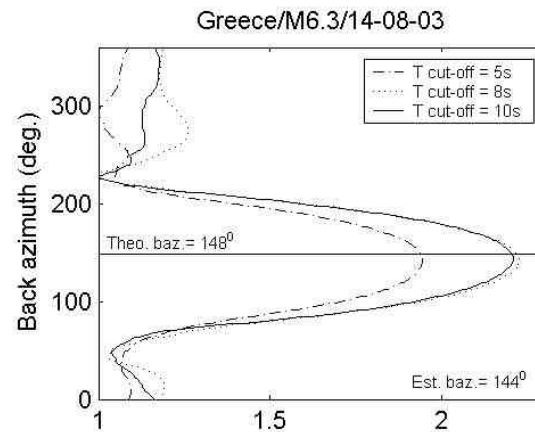
# Rotational signals in the P-coda? *azimuth dependence*





# Love waves - direction of energy

... from rotations and translations measured at one point ...

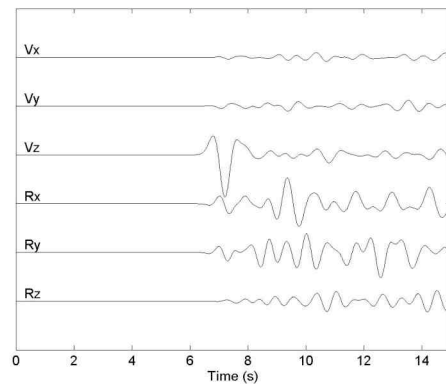
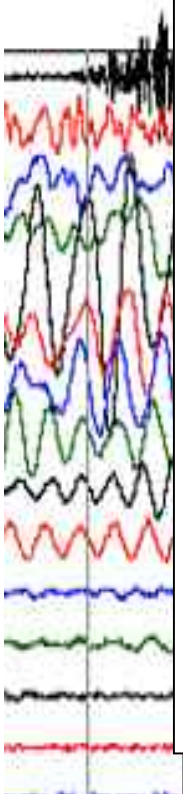


correlations in Love wave window



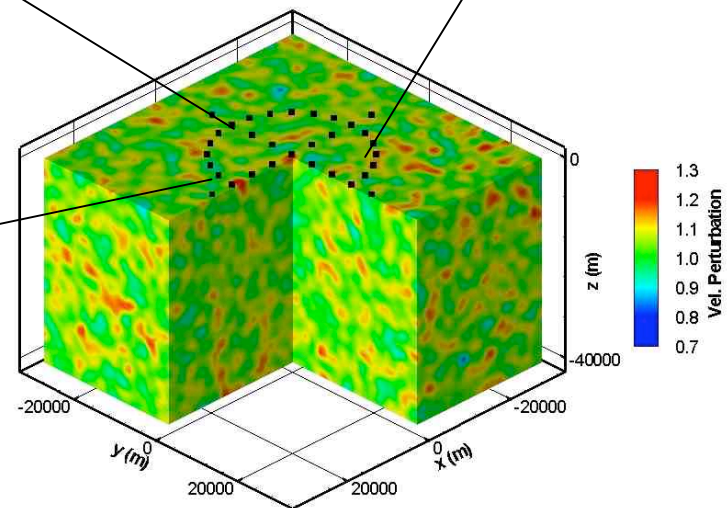
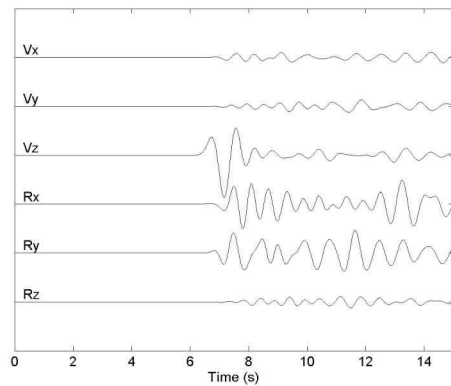
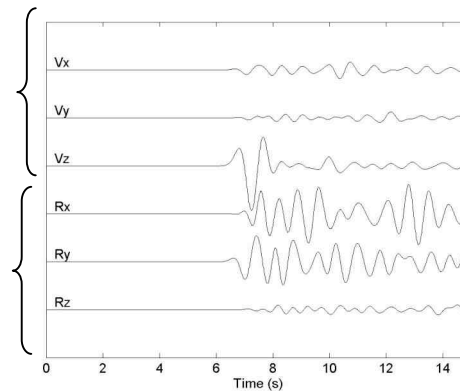


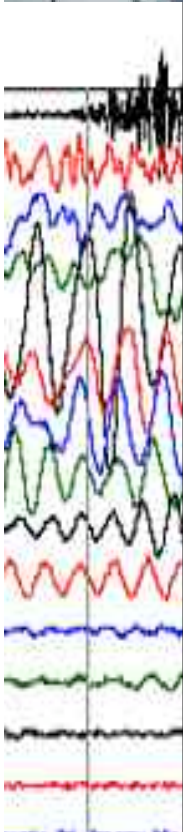
# P-SH scattering simulations with ADER-DG



translations

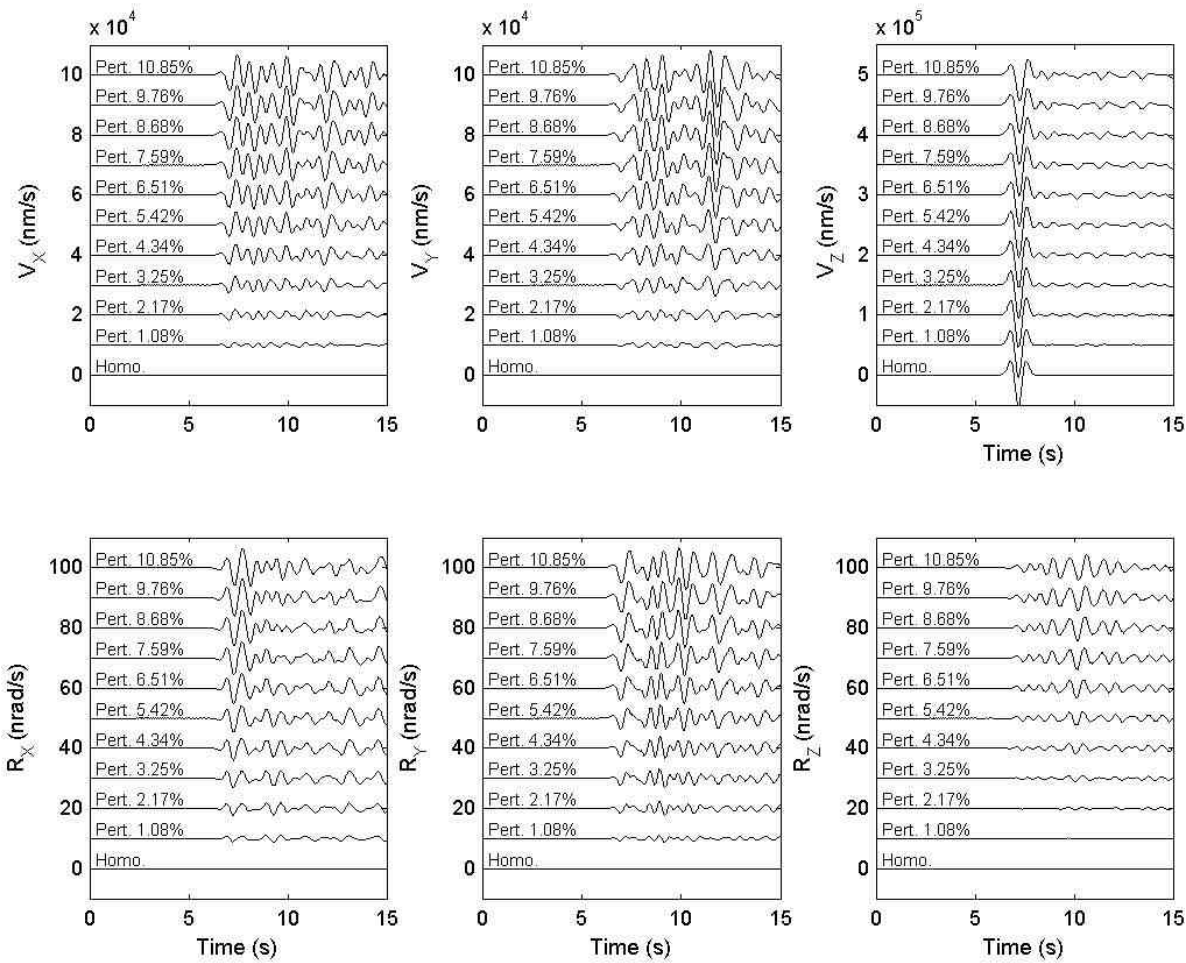
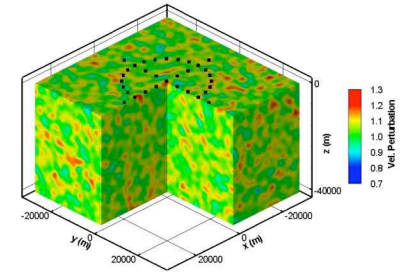
rotations



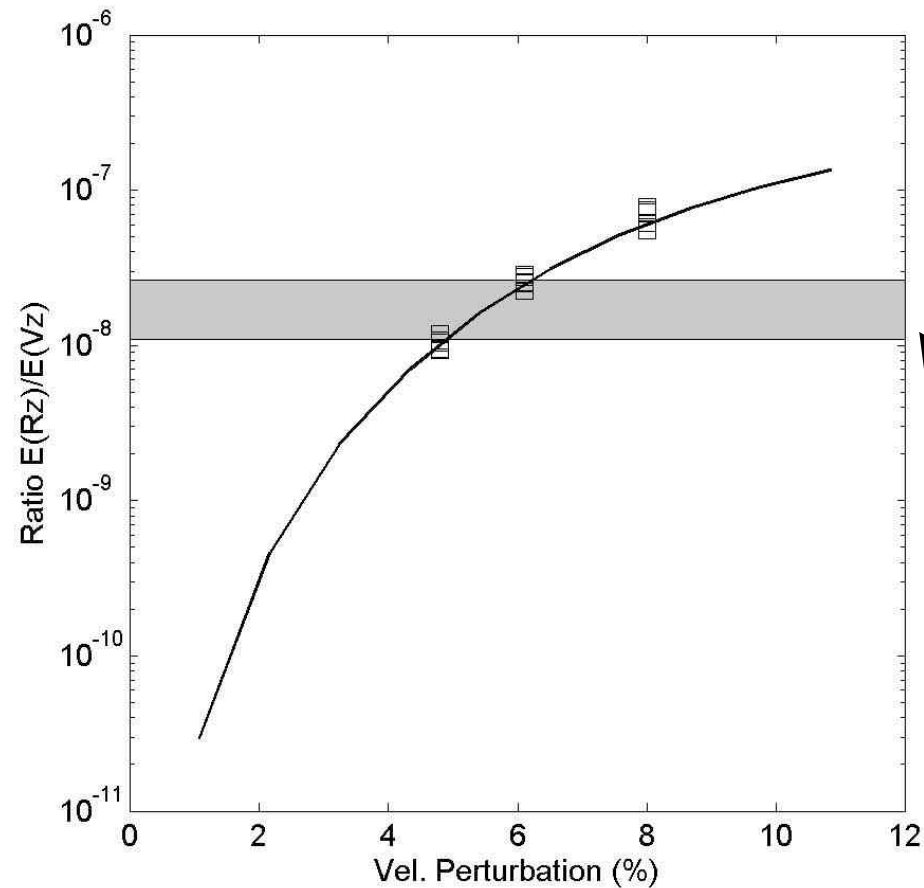
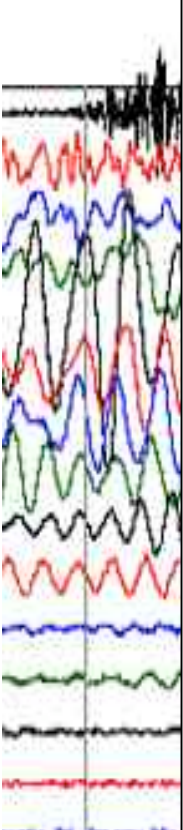


# P-SH scattering

*simulations with ADER-DG*



# Ratio of Rotational energy vs. Translational energy

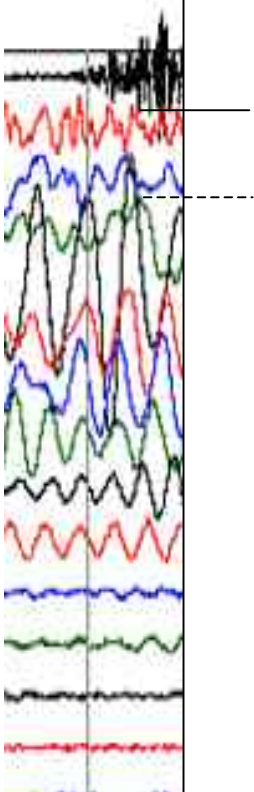


Observed  
range of  
energy ratios

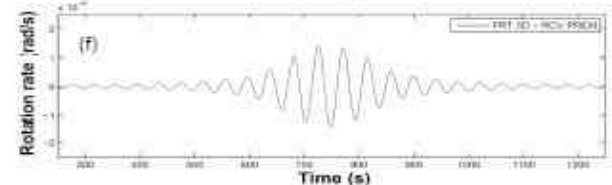
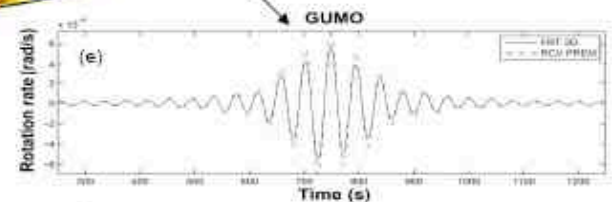
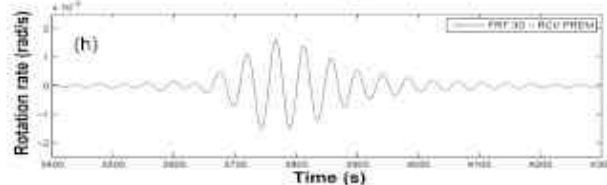
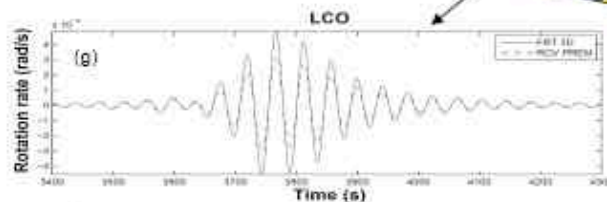
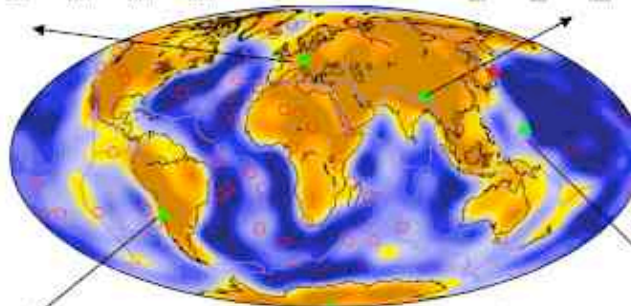
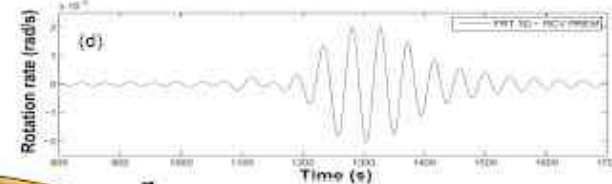
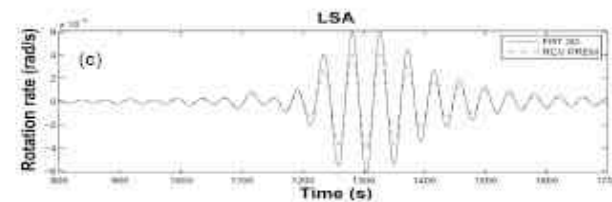
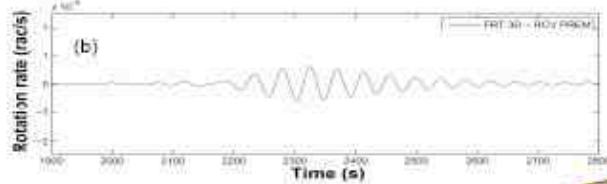
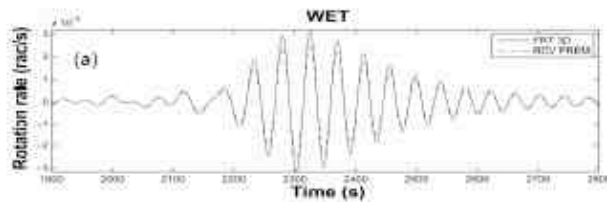


# Love wave wave rotations

JWKB synthetics



3D  
PREM

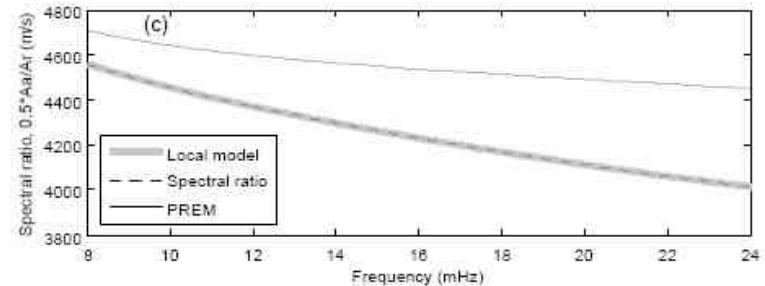
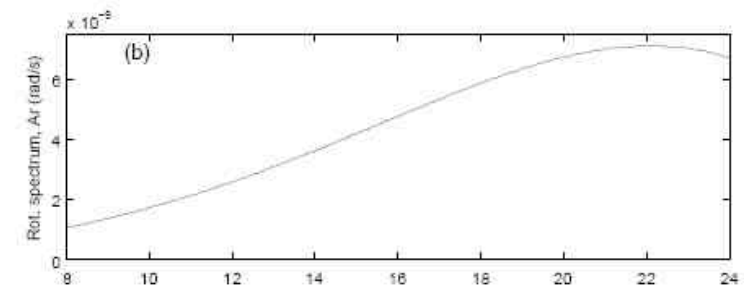
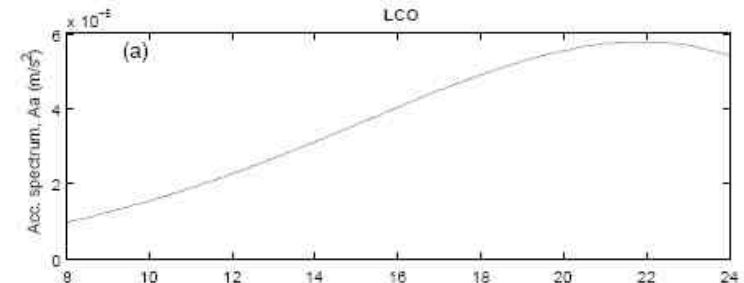
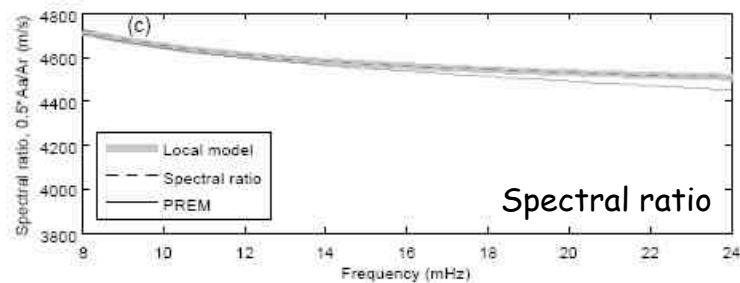
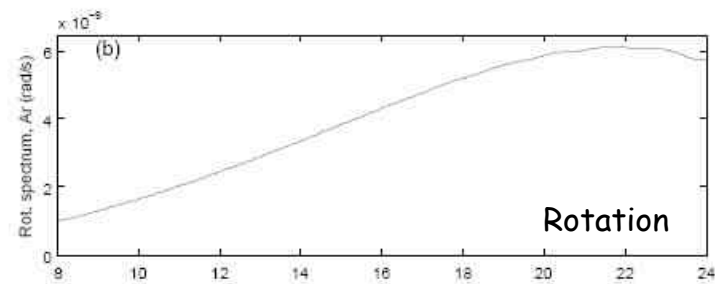
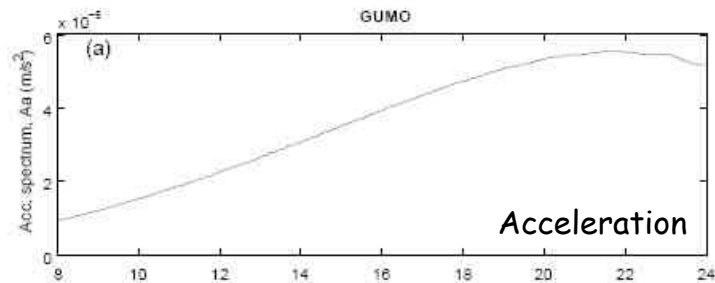


# Love wave dispersion

-> S-wave tomography

Oceanic Crust

Continental Crust



Ferreira and Igel., 2009, BSSA.