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REPORTS ON THE STATE OF SCIENCE, ETC.

Seismological Investigations.—*Twenty-third Report of the Committee, consisting of Professor H. H. TURNER (Chairman), Mr. J. J. SHAW (Secretary), Mr. C. VERNON BOYS, Dr. J. E. CROMBIE, Mr. HORACE DARWIN, Dr. C. DAVISON, Sir F. W. DYSON, Sir R. T. GLAZEBROOK, Professors C. G. KNOTT and H. LAMB, Sir J. LARMOR, Professors A. E. H. LOVE, H. M. MACDONALD, J. PERRY, and H. C. PLUMMER, Mr. W. E. PLUMMER, Professors R. A. SAMPSON and A. SCHUSTER, Sir NAPIER SHAW, Dr. G. T. WALKER, and Dr. G. W. WALKER.*

General.

OWING to the cancelling of the Cardiff meeting proposed for 1918 and to other reasons connected with the war, the present Report is made brief. It has been drawn up by the Chairman.

The Committee asks to be reappointed with a grant of 100*l.* (including printing), in addition to 100*l.* from the Caird Fund already voted. The grant was formerly 60*l.*, with 70*l.* for printing; 130*l.* in all; but during the war it has been reduced to 100*l.*, partly to meet the need for economy, partly because the printing has necessarily been less. The Government Grant Fund administered by the Royal Society has voted a subsidy of 200*l.* for 1918 as in recent years. With the above modification the budget remains practically the same as given in the Twentieth Report.

Owing to business reasons connected with the war, Mr. J. H. Burgess found it necessary to leave the Isle of Wight at the end of March, 1918. For half a dozen years he had devoted a considerable portion of his time (nominally one-half, but this was often exceeded in his enthusiasm for seismology) to the work at Shide. The Committee is greatly indebted to him for his prompt and valuable help when the sudden death of John Milne in 1913 left them in face of a threatened break in the work. Thanks to the special knowledge Mr. Burgess had acquired in working with Professor Milne, discontinuity was avoided, and whatever changes are found necessary in the future can be made with full consideration.

Mr. S. W. Pring remains at Shide for the present, though it is impossible to predict the length of his stay under present war conditions, which have already somewhat reduced the time he is able to devote (chiefly in the evenings) to seismology. His knowledge of Russian, which originally brought him into contact with Professor Milne, has been an important asset throughout, and has recently proved specially valuable in supplying translations of papers otherwise inaccessible.

The routine work generally is now in the hands of Miss Caws, who had been trained in it by Mr. Burgess. As a safeguard against her illness, and to enable her to take an occasional holiday, Miss E. F. Bellamy, of

the Oxford University Observatory, has been instructed by Miss Caws in the work, and undertook the routine observations during the absence of Miss Caws from the middle of July to the middle of August. It is proposed to set up at least one Milne-Shaw seismograph at Oxford under Miss Bellamy's care. Mr. James Walker has kindly placed at the disposal of the Committee the basement of the Clarendon Laboratory in which Mr. C. V. Boys made his gravity determination. The suitability of Oxford as an observing site can thus be tested under good conditions, and the results of the tests in a basement of that character may have an even wider application when plans for the future come to be discussed.

Finally, since the departure of Mr. Burgess naturally reduces the working power of the staff, Miss Bellamy has been definitely transferred to the seismological service for the present. This transference was approved by the Board of Visitors of the Oxford University Observatory 'as a provisional and possibly temporary arrangement' which 'is not to prejudice the resumption of her position as Assistant [in the University Observatory] at the conclusion of the experiment, should that be her wish.' The nature of the experiment here mentioned has perhaps been sufficiently indicated, but a few words of explanation may be added for the sake of clearness.

It was mentioned in last year's Report that the question of the future of seismology had attracted the attention of more than one body. Besides our own Committee, the Geodetic Committee of Section A of our Association, in discussing plans for a Geodetic Institute, were led to consider the possible association, in such an institute, of other branches of Geophysics with Geodesy. It was further mentioned that the Geodetic Committee had thereupon been suitably enlarged for the purposes of this discussion; and it should perhaps have been added that they had been invited by the Conjoint Board of Scientific Societies to act as their Committee also, and to report to them as well as to the British Association—and had, with the approval of the Council of the British Association, accepted that invitation.

Their report was in favour of the association of the three branches Geodesy, Seismology, and Tides in one Geophysical Institute. It was generally approved by the Conjoint Board, who appointed a small Executive Committee to formulate definite plans.

The transference of some of the Shide work to Oxford during the next year or two is definitely not intended to prejudice, or to embarrass in any way, the discussion of these plans. But since we are in several respects somewhat in the dark (*e.g.*, as to the precise value of a basement as a site for instruments, of a University as a centre of organisation, &c.), the transference will provide experience which may possibly be of some assistance to the discussion. Meanwhile, it is also the simplest solution of the question of ways and means at the present rather difficult time.

Instrumental.

The wireless time-signals have been received at Shide regularly, with some interruptions owing to derangement of the apparatus. The transit lent by the Royal Astronomical Society has been in reserve, but not much used during the year.

Towards the end of 1917 the temporary use of a 'dug-out,' constructed

for military purposes, was offered to Mr. Burgess, and the opportunity was taken to make some experiments. With Mr. Shaw's help the Milne-Burgess machine was dismounted from its pier at Shide, and remounted in the dug-out some few miles away. At some considerable personal trouble and inconvenience Mr. Burgess kept the machine running for a few weeks. [It was necessary to keep a pump going to avoid having the dug-out flooded.] No very satisfactory results were obtained, the wandering of the trace being just as serious as at Shide, and the experiments came to a natural end when the dug-out was reclaimed for military purposes. The main conclusion which emerged from the experiments was that rain had more to do with the wandering of the trace than had temperature. After rain the lines became much congested: when the rain ceased they were unusually expanded, returning to the normal separation when the weather had been fine for a few days. The inference seems to be that the wetting and drying of the ground caused slow movements in opposite directions—a conclusion in accord with previous experience at Shide. The dug-out was in a clay soil, which doubtless emphasised the effects.

Milne-Shaw Seismographs.

Several of these machines are nearing completion; that intended for Oxford is finished, and will be erected as soon as possible.

Suggested Corrections to Adopted Tables.

The work of disentangling the corrections to the adopted tables from other errors is attended with considerable difficulties, but nothing is more important, if the phases are to be rightly identified; and although progress is slow, the ground is being steadily cleared. One set of difficulties arises from ignorance of the clock errors, which may be expected especially at outlying stations. Recent experience aroused the suspicion that these are often considerable, so that accurate intervals S—P are attended by unexpectedly large errors in S and P separately. The experience here referred to is derived from the re-reduction of a number of the best observed earthquakes after applying the corrections to tables deduced in the last report, and carefully correcting the position of epicentre.

Hence attention was recalled specially to the investigation in the Introduction to 'The Large Earthquakes of 1913,' p. iii, which was confined to the use of intervals S—P, free from clock errors. This investigation gives no information about S and P separately which will enable us to correct the tables, and was naturally followed by a more comprehensive analysis where S and P were kept separate, the results of which were given in the Twenty-second Report, Table II. We may denote these two investigations by the symbols (1913) and (22) respectively.

Now for the smaller values of Δ (1913) differs essentially from (22), as is seen from Table I. below, and we must determine which is nearer the truth. For this the following method suggested itself.

Pairs of Stations on Opposite sides of the Epicentre.

Any pair of observing stations on directly opposite sides of the epicentre give a check on the values of S—P. Suppose for simplicity we

had two stations, A and Z, at ends of a diameter of the earth, and the epicentre E lay midway between them, so that $EA=EZ=90^\circ$, for which the tables give $S-P=658$ seconds. We are suggesting that this ought to be 658s.—17s.=641 seconds. If we are right, then 641 seconds will be observed at both A and Z. In proceeding to determine the epicentre we may use A alone (for in practice A may not be a single station but the whole body of European stations), and using our erroneous tables we should put the epicentre only $86^\circ.8$ from A, and therefore $93^\circ.2$ from Z; so that the observed 641 seconds at Z would be doubly in defect, since for $93^\circ.2$ the tables give 674 seconds. However we determine the epicentre (provided Δ_1 and Δ_2 are not too unequal), this double defect of 33s. or 34s. will be shared between A and Z. Two such antipodal stations are specially valuable because we cannot alter the sum of the two distances ($\Delta_A + \Delta_Z$) by changing the azimuth of the epicentre; and even when the antipodal condition is not quite fulfilled (as, say, for Pulkovo and Riverview, which are about 140° apart) a change of azimuth makes very little change in $\Delta_A + \Delta_Z$. For pairs of stations closer to the epicentre, the azimuth is often well determined by other stations; and unless they are very close to the epicentre the azimuth error only affects ($\Delta_1 + \Delta_2$) to the second order. Hence we have a good check on the values of $S-P$. The following satisfactory instances were collected from the published results of 1913, 1914, 1915. They are worth giving in detail to show the good accordance.

TABLE I.

Pairs of Stations on opposite sides of the Epicentre.

Δ_1	Δ_2	Mean	(S-P) ₁	(S-P) ₂	Mean
°	°	°	s.	s.	s.
6.0	6.9	6.4	+39	-10	+14
7.0	18.7	12.8	0	+7	+4
5.9	19.9	12.9	+22	+6	+14
5.9	20.4	13.2	+7	0	+4
7.8	20.0	13.9	+12	+26	+19
9.9	19.4	14.6	+30	-2	+14
9.9	22.3	16.1	+4	+8	+6
18.6	23.1	20.8	+1	-21	-10
14.8	30.0	22.4	+14	-10	+2
22.8	36.2	29.5	-3	-5	-4
26.3	36.8	31.6	-1	-4	-2
20.8	53.1	37.0	+8	-21	-6
25.9	49.0	37.4	-7	+4	-2
26.2	78.4	52.3	+2	-9	-4
40.2	69.5	54.8	-32	+15	-8
47.8	86.3	67.0	-9	-21	-15
47.2	87.0	67.1	-20	-10	-15
46.0	88.2	67.1	-20	-24	-22
58.6	79.1	68.8	-27	-10	-18
50.3	88.5	69.4	-8	-12	-10
48.9	90.6	69.8	-13	-3	-8
65.6	75.1	70.3	-12	-15	-14
68.5	73.5	71.0	-12	-10	-11
43.6	90.9	72.6	-8	-11	-10
63.9	81.4	72.6	-1	-18	-9
66.1	81.6	73.8	+3	-14	-6

Forming groups of these we obtain the mean values of Table II. :—

TABLE II.
Suggested Corrections to Tabular S — P.

No. in Group	Mean Δ		Mean Correction	
	(1913)	(22)	(1913)	(22)
1	6.4	+14	+ 4	- 4
4	13.2	+10	+ 8	- 8
4	18.5	+ 3	+ 3	- 8
4	33.9	- 4	-10	-13
2	53.6	- 6	- 7	- 6
6	68.2	-15	- 4	- 6
5	72.1	-10	+ 1	- 8

We infer that for the smaller values of Δ at any rate the investigation (1913), in which only S—P was used, is to be preferred to (22), in which S and P were treated separately.

Observations of PR₁ and SR₁.

But a knowledge of S—P will not suffice to correct the separate tables. We must find some other way of doing this, and recourse was had to the observations of PR₁ and SR₁.

We shall first assume that these represent reflections at the mid-point without loss of time. Thus PR₁ is a P wave reflected at the mid-point (M) between the epicentre (E) and the observing station (O) so that

$$EO=2EM=\Delta,$$

and the time for PR₁ is twice that for EM or $\Delta/2$.

In Table III. the first column gives Δ and the second gives the observed value of $\frac{1}{2}(S+P)-PR_1$, or as we may write it $\frac{1}{2}(S-P)+P-PR_1$, which is nearly constant. The observations were collected from the records

TABLE III.

Δ	Obsd.		Tables		O—C ₁		O—C ₂	
	s.	s.	s.	s.	s.	s.	s.	s.
23	77	98	-21	-15				
28	75	102	-27	-14				
33	80	103	-23	- 9				
38	83	100	-17	- 2				
43	88	95	- 7	- 5				
48	91	88	+ 3	+10				
53	95	85	+10	+10				
58	98	83	+15	+10				
63	102	82	+20	+ 8				
68	106	83	+23	+11				
73	109	87	+22	+11				
78	109	92	+17	+ 9				
83	106	92	+14	+ 9				
88	105	96	+ 9	+ 7				
93	102	101	+ 1	+ 4				

of several observations, especially Pulkovo, in the years 1914 and 1915, and are fairly numerous for all but the small values of Δ . The actual

figures given in the table were read from a smoothed curve. In the third column is given the corresponding quantity calculated from the adopted tables in use, followed in the fourth column by the differences $O-C_1$. In the next column $O-C_2$ is shown the effect of correcting these tables as in Table II. of the 22nd Report. It is seen at a glance that the tables are considerably in error, and that the suggested corrections reduce the errors. But before deciding to attempt any further reduction let us turn to the observations of SR_1 , of which the following were collected :—

TABLE IV.
Observed Values of SR_1 .

S-P s.	SR_1-S s.	Diff. s.	S-P s.	SR_1-S s.	Diff. s.
363	149	214	574	282	292
366	146	220	578	291	287
372	162	210	583	297	286
397	195	202			
(374)	(163)	(211)	(578)	(290)	(288)
525	234	291	606	332	274
530	249	281	634	341	293
541	266	275	647	382	265
542	263	279	673	357	316
550	257	293			
(538)	(254)	(284)	(640)	(353)	(287)

The records of SR_1 are often given to tenths of a minute only, suggesting some uncertainty; more observations are desirable, but those above quoted are consistent in showing that the difference $(S-P) - (SR_1-S)$ or $2S-P-SR_1$ is sensibly constant from (say) $S-P = 520$ seconds to 680 seconds, which may serve as a useful check.

Comparing the means (in brackets) with the adopted tables, we get the differences $O-C$, while the corrections of the 22nd Report give the column $O-C_2$.

TABLE V.
 $2S-P-SR_1$.

S-P s.	Δ o.	Calc. s.	Obs. s.	$O-C_1$ s.	$O-C_2$ s.
374	40.3	212	211	- 1	+ 6
538	68.0	220	284	+64	+33
578	75.2	241	288	+47	+21
640	86.5	272	287	+15	+ 9

We again see at a glance that the adopted tables are sensibly in error, and that the corrections found in the 22nd Report, while they reduce these errors, do not annihilate them. We must proceed further in the same direction.

It is easily seen that the further alterations must be chiefly in SR_1 , and not in S (or P). Thus let us fix attention on the worst case, where $S-P=538s$. For this value of $S-P$ the adopted tables give $\Delta=68^{\circ}0$; but when we correct the tables we get $\Delta=69^{\circ}2$. Note that $S-P$ is

the observed quantity, and Δ is only inferred. But the consequent change in $(S-P)+(S-SR_1)$ is small and in the wrong direction; for though S is increased by 14 seconds, SR_1 is increased by 18 seconds. After altering Δ we have, however, still to apply the errors of S and of SR_1 . The complete process may be represented thus, when the corrections used are those of the 22nd Report:—

	$S-P$ s.	Δ	$\frac{1}{2}\Delta$	$+S$ s.	$-SR_1$ s.	$(S-P)+(S-SR_1)$ s.
Tables	538	68 ⁰ 0	34 ⁰ 0	+1202	-1520	+220
Δ	-	+1.2	+0.6	+ 14	- 18	- 4
Errors	.			- 3	+ 38	+ 35

We see at a glance that the efficient corrective is the error in SR_1 which is taken to be twice that of S at $\Delta/2$. The error of S at $34^{\circ}6$ is found in the 22nd Report to be -19 seconds; if this were only larger numerically, say -34 seconds, we could completely destroy the + 64s. of Table V., column O-C₁.

The observations of SR_1 are not, however, so numerous as those of PR_1 to which we now return. Let us neglect the small change in Δ . Beginning at the bottom of the table, let us assume that we know that the error at $\Delta=93^{\circ}$ is -7 seconds, from the 22nd Report. Then, since

$$P-PR_1 = +1s.$$

$$PR_1 = P-1s. = -7s. -1s. = -8s.$$

i.e. the error at $\Delta = 46^{\circ}5 = -4s.$, which so far accords quite well with that deduced independently in the same Table II. of the 22nd Report. But as we proceed upwards we shall not find the same good accordance. The errors for the smaller values of Δ deduced in this way from PR_1 are as in Table VI. :—

TABLE VI.

Errors of P deduced from PR_1 .

Δ	From PR_1	From (22) Old Tables δP_1	New		Possible δP_3	Corrn.	
			s.	s.		s.	s.
11.5	+10	-1	172	182	184	s.	s.
			34	35	36		+12
14.0	+11	+1	206	217	220		+14
			33	29	31		
16.5	+ 7	+2	239	246	251		+12
			30	27	25		
19.0	+ 4	+2	269	273	276		+ 7
			30	26	18		
21.5	0	+1	299	299	294		- 5
			29	26	21		
24.0	- 3	0	328	325	315		-13
			25	24	25		
26.5	- 4	-1	353	349	340		-13
			25	23	26		
29.0	- 6	-3	378	372	366		-12
			24	22	25		
31.5	- 8	-4	402	394	391		-11
			23	21	24		
34.0	-10	-6	425	415	415		-10
			21	21	22		

TABLE VI.—*continued.*
Errors of P deduced from PR₁.

Δ	From PR ₁	From (22) Old Tables δP		New	Possible δP_3	Corrn.
36.5	-10	-6	446	436	437	- 9
			20	21	21	
39.0	- 9	-7	466	457	458	- 8
			21	22	21	
41.5	- 8	-7	487	479	479	- 8
			19	20	20	
44.0	- 7	-6	506	499	499	- 7
			18	21	20	
46.5	- 4	-4	524	520	519	- 5

We see that the investigation of the 22nd Report (shown in the third column of Table VI.) fails for the smaller values of Δ in much the same way as is manifested in Table II. above; viz. from $\Delta = 10^\circ$ to $\Delta = 20^\circ$ the errors are not sufficiently positive if we may accept the indications of PR₁, while near $\Delta = 35^\circ$ they are not sufficiently negative. The column deduced from PR₁ starts with a hill and falls into a valley; in the investigation (22) the hill is cut down and the valley filled up; and it is reasonable to attribute these modifications to the effects of compromise due to the determination of the epicentre with the faulty tables.

In the fourth and succeeding columns of Table VI. the effects of applying the newly-suggested corrections are shown. First, under the heading 'Old Tables' are given the tabular values for P, followed by the differences δP_1 . Under the heading 'New' are given the tables with corrections as from PR₁ and the differences δP_2 . Now it is seen that these differences δP_2 are approximately constant from $\Delta = 29^\circ 0$ to $\Delta = 46^\circ 5$, whereas δP_1 drops steadily. We are reminded of the phenomenon shown by the Pulkovo angles of emergence. (See p. 54 of Dr. G. W. Walker's 'Modern Seismology'). In the last report Dr. Walker recalled attention to these observations, emphasising Galitzin's belief in them, and suggesting in explanation a focal depth of the order of 1,300 km. No support for this hypothesis has been otherwise forthcoming, and it seems possible that a simpler explanation may be found in errors of the tables; for a comparatively slight *further* modification would give angles of emergence in fair accord with the Pulkovo observations. Such a modification is shown in the column of Table VI. headed 'possible.' Its differences δP_3 show a fall to $\Delta = 20^\circ$, then a rise to about $\Delta = 28^\circ$, and then a fall again, which is all that is required. We proceed to show that this possibility is independently suggested by a study of the Y phenomenon or polychord.

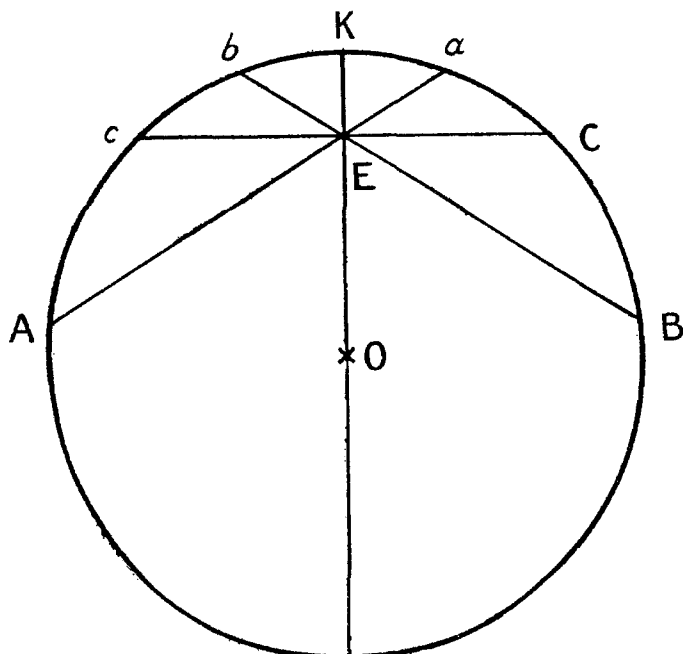
The Polychord or Y Phenomenon.

Is it simply PR₅?

In the 21st Report the following approximate times were deduced from the earthquake of 1913, March 14, for the Y phenomenon:—

	$\Delta = 95^\circ$	100°	105°	110°
Time for Y	= 1420s.	1470s.	1490s.	1520s.
Average vel.	14.9s.	14.7s.	14.2s.	13.8s.

The average velocity, added in the last line, falls off very definitely, and the hypothesis of a 'polychordal' P wave transmitted by numerous reflections with approximately uniform arcual velocity was therefore withdrawn. But in the course of the examination of Dr. G. W. Walker's hypothesis of deep-seated focus a modification of the hypothesis suggested itself which seems worth attention. Repeating the subjoined figure from the 20th Report,



we shall assume that cC is an arc of $17^{\circ}.1$ so that a P wave starting from a deep-seated focus E along EC will be reflected at C so as to strike the surface again at $C_1 C_2 C_3 \dots$ where $CC_1 = C_1C_2 = C_2C_3 \dots = 17^{\circ}.1$; and after five such reflections will reach a distance Δ from K (the epicentre) equal to $5.5 \times 17^{\circ}.1 = 94^{\circ}$. The whole time required would be, with adopted tables,

$$5.5 \times 246s. = 1353s.,$$

but if we adopt the corrections of Table VI. as in the column 'New' we should get

$$5.5 \times 253s. = 1391s.$$

The value extrapolated from the above-quoted values for Y would be still greater, viz. 1410s., but we see at once that a comparatively slight increase to the tables would give this exact value: viz. the time for an arc of $17^{\circ}.1$ must be 256s., indicating a correction to tables of + 10s. near $\Delta = 17^{\circ}$ instead of + 7s.

For paths slightly differing from EC , such as Ea or EB , the chords are greater, and five reflections will bring us beyond 94° in either case; in

fact, there can be no PR_5 when Δ is much less than 94° (for the actual limit see below), which accords with the facts of the quake of 1913, March 14, from which the above figures were deduced in the 21st Report, if we exclude a doubtful Pulkovo observation. This limiting value for Δ gives us in fact the arc $17^\circ.1$, and consequently the depth of focus. If EC be a straight line we find $EK = 0.011$ of the radius; say 44 miles or 70 km. These figures apply only to this particular earthquake, and will also require modification if the path cC is curved; but they give the order of quantity suggested.

As we pass to greater values of Δ the wave must start along Ea or EB . If it starts along Ea , the total length of path will be less than $5.5aA$, and cannot be so small as $5.0aA$. If it starts along EB the length will exceed $5.5bB$ or $5.5aA$.

Thus $\Delta=110^\circ$ may be reached in two ways: either by (let us say for illustration) 5 reflections or 6. The times of transmission would be (using column 'New' of Table VI.) :—

$$5 \times \text{time for } 22^\circ = 5 \times 304s. = 1520 \text{ seconds,}$$

$$6 \times \text{time for } 18.3^\circ = 6 \times 266 = 1596 \text{ seconds,}$$

and since we observe the *earliest* disturbance we fix attention on Ea and neglect EB .

Assuming the paths of the waves to be straight lines as in the figure, let cC subtend an angle $2C$ at the centre; and let aA , inclined to cC at an angle ϕ , subtend an angle $2A$ at the centre. Then

$$\cos A = \cos C \cos \phi = \cos 8^\circ.55 \cos \phi$$

$$\text{arc } Ka = A - \phi.$$

In Table VII. are given, for various values of ϕ , first the arcs $2A$ and $A - \phi$, then the whole arcs $10A + (A - \phi)$; then the estimated times for Y from the figures quoted above from the 21st Report. From these we subtract the time for the arc $A - \phi$, which we can estimate only approximately by taking it as

$$\text{arc in degrees} \times 14.9s.$$

The remainder, divided by 5, gives the time for the arc $2A$, below which is subjoined the present tabular value and the necessary correction, followed by the correction deduced from PR_1 in Table VI.

TABLE VII.

	°	°	°	°	°
ϕ	= 0.0	3.0	5.0	6.0	7.0
$2A$	= 17.1	18.0	19.8	20.8	22.0
$A - \phi$	= 8.6	6.0	4.9	4.4	4.0
Whole arc	= 94.0	96.0	103.9	108.4	114.0
	s.	s.	s.	s.	s.
Whole time	= 1410	1430	1485	1512	1548
Time of $(A - \phi)$	= 128	89	73	66	60
Time of $2A$	= 256	268	292	289	298
Tables	= 246	257	279	291	305
Correction	= +10	+11	+ 3	- 2	- 7
Table VI.	= + 6	+ 5	+ 3	+ 1	- 1

The last two lines are not *very* different, and we notice that the corrections now suggested are generally similar in form to those formerly

found, but greater in amount. As has been remarked above, the existence of a hill and valley in the curve, which compromise has obliterated, is quite a fair possibility.

The question, however, whether the observations can be fitted to so considerable a hill and valley can only be answered after laborious re-reduction; such examination as has been made is not unpromising, but is embarrassed by the uncertainties of clock error already referred to. A glance at the records, for instance, of 1913, January 27, will show the doubtful nature of the records near the epicentre.

The explanation above suggested brings with it a number of questions. Firstly, why should PR_5 be specially prominent, compared, say, with PR_4 or PR_6 ? The answer is that PR_5 happens to fall in a place where it can be mistaken for S, while the others do not. But the suggestion of a sensible focal depth necessitates the re-investigation of other reflected waves, especially of PR_1 . We notice two important points.

Firstly, the wave which starts along EC reaches the surface as PR_1 at $\Delta = 8^\circ.6 + 17^\circ.1 = 25^\circ.7$; and it might at first sight appear that we cannot have any PR_1 for a value of Δ less than this, for the arcs 2A and 2B are both greater than 2C, as already remarked. But a wave starting along Ea reaches the surface after one reflection at

$$\Delta = 2A + A - \phi,$$

and from Table VII. we see that the sum of these quantities diminishes at first, though it ultimately increases. The minimum value is given by the condition

$$d\phi = 3dA,$$

which leads to

$$(3^2 - 1) \sin^2 \phi = \tan^2 C$$

whence $\phi = 3^\circ, \Delta = 24^\circ.$

Within $\Delta = 24^\circ$, then, no PR_1 can be received.

Now, the evidence is distinctly in favour of the existence of some inferior limit for Δ of this kind. Within $\Delta = 24^\circ$ the records are few and discordant, and the lowest value of $S - P_1$ hitherto found for which PR_1 has been recorded is 204 seconds, corresponding to $\Delta = 18^\circ$, or a focal depth of, say, 25 miles. A more extended investigation of this point must, however, be deferred.

The second point is that at the point C (where $\Delta = 25^\circ.7$) PR_1 arrives not by means of two equal arcs of 13° , but by unequal arcs of $8^\circ.6$ and $17^\circ.1$. Hence the tabular computation of Table VI. becomes unsuitable. Using the adopted tables we should get for $\Delta = 26^\circ$ by equal arcs of 13° , $\frac{1}{2}(S+P) - PR_1 = 99$ seconds, by arcs of $8^\circ.7$ and $17^\circ.3$ $\frac{1}{2}(S+P_1) = 105$ seconds. Hence the large negative values of O-C, at the head of Table III., should be still further increased; and, moreover, when we deduce from them corrections to P the dividing factor is 1.5 instead of 2.0, and the corrections apply to larger values of Δ , all of which changes emphasise the positive corrections to P for small values of Δ . As above remarked, however, EC does not give the minimum PR_1 , which is for $\Delta = 24^\circ$, by arcs of 6° and of 18° , $\frac{1}{2}(S+P) - PR_1 = 107$ seconds, the computation by equal arcs giving 98 seconds.

For greater values of Δ PR_1 may come by either of two paths. It is easy to take from a diagram the alternatives as follows:—

	o	o	o	o	o	o	o
Δ	= 24	25	26	27	28	29	30
Arc $(A-\phi)$	6.0	4.5	4.0	3.6	3.3	3.0	2.7
Long arc	18.0	20.5	22.0	23.4	24.7	26.0	27.3
	s.	s.	s.	s.	s.	s.	s.
$\frac{1}{2}(S+P)-PR_1$	107	113	118	122	127	133	138
	o	o	o	o	o	o	o
Arc $(A+\phi)$	6.0	8.0	8.9	9.7	10.5	11.3	12.0
Long arc	18.0	17.0	17.1	17.3	17.5	17.7	18.0
	s.	s.	s.	s.	s.	s.	s.
$\frac{1}{2}(S+P)-PR_1$	107	104	104	105	106	107	150

Here it certainly looks as though the *observed* PR_1 starts with the longer arc $A+\phi$ instead of the short arc $A-\phi$ as we have assumed for PR_5 ; for only by this supposition can we obtain approximate constancy for the quantity $\frac{1}{2}(S+P)-PR_1$. There is nothing unreasonable in this difference between the two cases: for PR_5 , if we are right in identifying it with Y , is usually read for S ; we read the *first big* movement, and, as already remarked, the PR_5 by $A+\phi$ follows that by $A-\phi$. Hence the former is read. But in looking for PR_1 we should naturally take a movement which is not too near P . The PR_1 which starts with $A-\phi$ runs up closer and closer to P as ϕ increases.

We have, however, still to explain why $(S+P)/2-PR_1$ should be about 105s. instead of about 80s., as at the head of the 'observed' column in Table III. On the present hypothesis the tables are wrong to this extent. When $\Delta=28^\circ$, for instance, if we take the *last* column of Table VI. the error of P is $-12s.$ and that of S should be nearly double, say $-20s.$ which gives for $(S+P)/2$ an error of $-16s.$ and PR_1 is in error by $2 \times 14 = 28s.$; making altogether 44s., which reduces the 106s. of the tables to 62s., as compared with 75s. observed. These corrections are apparently *too large*, and it may be readily admitted that the last column of Table VI. probably goes too far. If we use the first column we get, say, $-8s.$ for $(S+P)/2$, and $-22s.$ for PR_1 , which is just the quantity required.

Somewhat similar considerations apply to PR_2 . The whole arc is now

$$A - \phi + 4A,$$

and becomes a minimum when

$$d\phi = 5dA,$$

which leads to

$$(5^2 - 1) \sin^2 \phi = \tan^2 C,$$

so that $\phi = 1^\circ.7$, $A = 8^\circ.75$; minimum $\Delta = 42^\circ$.

Generally, for PR_n , the corresponding equation is

$$\{(2n+1)^2 - 1\} \sin^2 \phi = \tan^2 C,$$

or $(2n+1)\phi = C$ approximately; so that when $n=5$, $\phi = 0^\circ.77$, $A = 8^\circ.58$; minimum arc being $93^\circ.6$, close to the value obtained by starting along EC .

Observations of PR_2 are fairly numerous beyond $S-P=542$ seconds (say $\Delta=69^\circ$), but only three have been found for smaller values, viz.:

$$S - P = 487s., \quad 363s., \quad 360s.$$

$$\text{Say } \Delta = 59^\circ, \quad 38^\circ.5, \quad 38^\circ.0,$$

according to adopted tables. Further scrutiny is of course desirable but the evidence is so far satisfactory.

Of PR_3 and PR_4 little more need be said at present than that they do not fall near enough to S to be confused with it, while PR_6 probably lies beyond S . The rough minima in Δ for their appearance are:

	PR_2	PR_3	PR_5	PR_6
$\Delta =$	60°	77°	94°	111°
	s.	s.	s.	s.
Time =	896	1152	1408	1664
S (Tables) =	1103	1309	1496	1657
S (corrected) =	1102	1300	1474 ?	1637 ?

The above notes are obviously tentative and incomplete, but they represent the result of a good deal of work by the method of trial and error, and may serve to show possibilities. Especially is it hoped that they may show the importance of an identification of the phenomena through more accurate tables of P and S , on which the main computing strength of the Shide organisation is being concentrated for the present. It may be that there are too many variable elements to make great accuracy possible, but experience of working with the residuals suggests the contrary view, which is at any rate worth thorough testing, even if the ultimate result be disappointing.